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Dynamics of Elastic Objects under Movable Inertial Loading - Some Features of the Mathematical Models and Analogies

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Abstract. This paper describes some features and analogies of the mathematical models for the elastic elements with movable load and for the elastic elements of changeable length. In these systems two forms of own oscillations - the own component and the accompanying one, displaced in phase to the right angle correspond to every frequency of the system. The accompanying component is caused by the mobile inertia load or by the changeable length and they are not trivial only when this factor exists. As for objects with time-varying length, these problems lie in outside of the scope classical problems of mathematical physics due to that the eigenfrequencies and eigenforms become time-dependent functions. This non-classical section of the mathematical physics is waiting for its development, new researches and generalizations.

Introduction

170 years have gone since the day of the first formulation of the problem of acting of movable loading on the elastic structure and building, after decay of Chester bridge in England in May 1847. During this time a lot of problems of dynamical impact of the movable loadings are different by their nature, behavior and influence on the elastic structures, systems, and buildings were been considered, solved and verified.

State-of-the-art of technology, increased road traffic intensity, intensification of flaw process indicate the use of more accurate mechanical and mathematical models, those more comprehensively and precisely reflect uncover essence of the phenomenon, that necessity improvement modification traditional and search for a new concepts and methods of researches

In the well-known review [7], dedicated to 100th anniversary of the problem formulation, the famous scientists in mechanical engineering Ya.G. Panovko wrote: “The problem of dynamic acting of movable load, 100th anniversary of which we have celebrated in 1947, by today did not lost their up-to-date status, the life still set new tasks and caused by this following motion of the theory to forward”

In agile XX-XXI centuries significant increasing of masses and velocities of motion sets new tasks, requires their solution, causes in its turn developing new approaches in the mechanical and mathematical modeling, new and improving of old methods of their research, that allow more comprehensively discover all quantitative and qualitative features of the kinematical and dynamical properties of the system motion.

Nowadays the keen interest to this problem arose due to intensively usage of information technologies, that allows to research the mathematical model and to analyze their results more deeply and comprehensive. The traditional representation of mechanical systems under movable inertia loads has been changed significantly.

The simple examples of those systems are bridges with moving vehicles, pipelines, bars, plates, envelopes loaded by the moving liquid or gas.

As the problem of this class we could treat some dynamical problems of the variable-length and time-dependent length objects, the dynamic problem of the objects under longitudinal motion, such as threads, wires, profile rod in the rolling process, strip and chain saw, belts of the belting, the cables of mining lifts and others.

The dynamical problem of moving variable-length objects of and objects in longitudinal motion such as threads, wires, profile rod

In dependency of the analytical model of inertia properties of the elastic structure and acting movable loading we could use the following four variants of statement of the problem of influence of the movable loading on the elastic structures and buildings [5,8]. The most complex for the practice is a fourth variant, that considers both inertia forces of the structure and inertia forces of the movable loading. Research on the qualitative and quantitative properties of the motion of such objects could be reduced to analysis of the following mathematical model

$$L\left(x, l, \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) w = L_1\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial t}\right) \cdot q(x,t) \quad (1)$$

where

$$q(x,t) = -\frac{q_0 + q_1}{g}\frac{\partial^2 w}{\partial t^2} - 2\frac{q_1 v}{g}\frac{\partial^2 w}{\partial t \partial x} - \frac{q_1 v^2}{g}\frac{\partial^2 w}{\partial x^2} \quad (2)$$

with the respective boundary and initial conditions, where under constant motion velocity

Features of the mathematical models of the elastic objects under movable loading

The main features of the mathematical models of such statements of the problem, at first is presence of the inertia operator q(x,t) in some of its form in the differential equation. It is distinguishing detail, that the force acting is dependent from the load intensity $q_1(x)$, the velocity v of the loading stream, the elastic strain w(x,t). Moreover, it is clearly relation between the force acting and the strain acceleration $w_{tt}(x,t)$, velocity of angular deformation $w_{tx}(x,t)$ and variation of curvature of the elastic line of the object $w_{xx}(x,t)$, that is in such systems the force acting is a following of the system behavior and changing its value and direction during the deformation.

Thereby, force acting applied to the elastic object caused by movable mass is not predefined and depends on the state of the system. It is the second feature of the dynamic problem of the elastic system in the inertia force field of movable loadings. The third significant features of these problems is that the mathematical model should contain the mixed derivative of odd-order by time in one of its form that represents Coriolis acceleration of the movable mass loading and does not allow to separate space and time variables by using the Fourier schema in the field of real functions.

An aerodynamic and hydrodynamic action of a liquid or a gas applied to the elastic object could be reduced to the same kind of the inertia operator. The velocity of the liquid stream in pipelines of the aircrafts is ranged between 50-80 m/s, and 200-250 m/s for gases and aircraft failures due to the instability of their pipelines attain 60% from the total number of failures [5,6].

The movable loading could be distributed uniformly or by some law, that could be discrete or continuously distributed with discrete inclusions with the constants or time-varying velocity. It is known an applied mathematical research has a successive approximation structure. At the beginning the rough approximation should be building, then the mechanical and the respective mathematical model or the technique of analysis of the mathematical model could be refined to get more accurate solution that could be amended at the next step as well.

The rough approximation has ancillary initial sense to get more accurate solution. The advantage of the rough approximate models and solutions is simplicity, transparency and evident, and accordingly to these the schema of applying of the solutions in most cases

Mechanical, mathematical models and some analogies

As it is known, Fourier method of mathematical physics allows to get solutions of some class of partial differential equations in the explicit form [4,5]. Only in relatively simple cases it is possible to build up the solutions of partial differential equations as a sum of particular solutions in the form of product of the separated functions.

To those equations belong the equations of (eigen) oscillations of the string, the beam and some others. The direct applying of this method to the dynamic problem of elastic systems under movable inertia loading is not possible in general cases.

That is why some authors tried to use this method by the way of its modifying and generalizing. One of the first publications was H.Steuding [10], where the lateral oscillations of the beam under movable distributed and concentrated loadings have been considered. The second one G.W.

Housner [9] proved that the general solution of the partial differential equation of elastic oscillations under movable inertia loading could be obtained as a linear combination of particular solutions, those contain symmetric and antisymmetric forced forms shifted by 90 degrees in their phase. Moreover, antisymmetric forced forms occurred due to the mixed derivative odd-order by time and Coriolis' inertia forces caused by movable loading and related through them to symmetrical forced forms. The symmetrical forced forms under non-movable loading are matched to the eigenforms of the loaded system.

Two above works began the method of two-wave representation of the elastic system oscillations under movable inertia loading and its physics interpretation had been provided by O.A. Goroshko [3,4].

Using the method of two-wave representation of the oscillations for research of those system, that allows in some cases to obtain analytical solutions, the general solution of the differential equations could be found as a sum of two infinity series the first series is a classical part of the solution and the second one is the part of the solution caused by presence of odd-order by time mixed derivative and inertia of movable loading, that could not be discovered by using of traditional direct methods of mathematical physics.

The forms of the first group are called as eigenforms and the forms from the second group are accompanying oscillation forms of an elastic system. Accompanying oscillations could be non-trivial if the elastic system is loaded by movable inertia loading.

The modes of the first group called eigenmodes, and the modes from the second one are called as accompanying modes of the elastic system oscillations. Accompanying modes are induced and non-trivial when the movable inertia loading is present

Today more penetrating and thorough research on the dynamic problem of elastic system under movable inertia loading by the method of two-wave representation is supported by modern information technologies, that was never used before, especially in the days of H. Steuding, G. W. Housner, Ya.G. Panovko and others.

Analogies of the mathematical models of the dynamics of elastic objects under movable loading and statics

As for analogues of mathematical models, it is easy to see, that the problem of lateral oscillation of the beam under uniformly distributed inertia loading in the critical mode could be reduced to the problem of solving of the differential equation [8]

$$EJ_{min} w^{IV}(x) = -m V_{cr}^2 w^{II}(x) , \tag{3}$$

with respective boundary conditions, and to get critical value of the compressing force for the beam, as it is known, could be reduced to the solving of the following differential equation

$$EJ_{min}\, w^{IV}(x) = -F_{cr} w^{II}(x) . \tag{4}$$

Analysis of these equations shows that the mathematical models of these problems are identical, that is to say some mathematical analogy exists and by using of this analogy we will get the approximate values of the critical speed of motion of the loading when the pinned beam will have buckling failure

$$F_{cr} = \frac{\pi^2 E I_{min}}{(\nu l)^2} = m V_{cr}^2 \text{ or } V_{cr} = \frac{\pi}{\nu \ell}\sqrt{\frac{E I_{min}}{m}} . \tag{5}$$

In the formulas given (3) - (5):

E – Young's modulus of longitudinal elasticity of the beam material;

w (x) - deflection of arbitrary cross-section of beam;

m - mass of unit of beam length;

I min - the axial moment of inertia of the cross-section;

F cr is the critical value of the compression force by Euler

V cr - the critical speed of the movable loading;

ν - coefficient of the effective length of the beam, which depends on the conditions of fixation of cross-sections

Considering the problem of lateral oscillations of the rectangular plate Lxb of thickness h with the movable stream of distributed load $q_1(x)$ moving along L side with speed v (Fig.1). At the first stage we assume that the speed of moving of the movable load is constant. As it is known the equations of equilibrium of the plate element under the lateral loading Z has the following representation [1,2]

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+Z=0\,, \tag{6}$$

where

$$M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right),\; M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\mu\frac{\partial^2 w}{\partial x^2}\right),\; M_{xy}=-D(1-\mu)\frac{\partial^2 w}{\partial x\partial y}\,. \tag{7}$$

After substitution of expression (7) in the differential equation (6) we could obtain the equation of equilibrium of the plate relative to the deflection function w(x,y)

$$D\frac{\partial^4 w}{\partial x^4}+2D\frac{\partial^4 w}{\partial x^2\partial y^2}+D\frac{\partial^4 w}{\partial y^4}=Z \tag{8}$$

where

$$Z(x,y,t)=-\frac{q_0+q_1}{g}\frac{\partial^2 w}{\partial t^2}-2\frac{q_1 v}{g}\frac{\partial^2 w}{\partial t\partial x}-\frac{q_1 v^2}{g}\frac{\partial^2 w}{\partial x^2}$$

is the effect from moving along axis x of the stream of distributed movable loading with constant velocity. In the expressions above h – thickness of the plate, E, μ – Young module and Poisson coefficient of plate's material, D – bending cylindrical stiffness of the plate.

Finally the problem about the lateral vibrations and buckling of the rectangular plate Lxb with thickness h and the stream of distributed loading that is moving collaterally to the longest side of the plate at the velocity of v could be reduced to solving of the differential equation

$$D\frac{\partial^4 w}{\partial x^4}+2D\frac{\partial^4 w}{\partial x^2\partial y^2}+D\frac{\partial^4 w}{\partial y^4}+\frac{q_0+q_1}{g}\frac{\partial^2 w}{\partial t^2}+2\frac{q_1 v}{g}\frac{\partial^2 w}{\partial t\partial x}+\frac{q_1 v^2}{g}\frac{\partial^2 w}{\partial x^2}=0\,, \tag{9}$$

where

q_0- distributed weight of the plate;

q_1- distributed weight of the movable loading stream;

D- cylindrical stiffness of the plate.

We will be getting the solution of the partial differential equation (9) for corresponding boundary and initial conditions. In the case when all edges of the plate are hinged

$$\begin{aligned}&w(0,y,t)=0,\quad w(L,y,t)=0,\quad w''(0,y,t)=0,\quad w''(L,y,t)=0,\\&w(x,0,t)=0,\quad w(x,b,t)=0,\quad w''(x,0,t)=0,\quad w''(x,b,t)=0.\end{aligned} \tag{10}$$

We could use the initial conditions as follows:

$$w(x,y,0)=f_1(x,y),\quad \frac{\partial w(x,y,0)}{\partial t}=f_2(x,y) \tag{11}$$

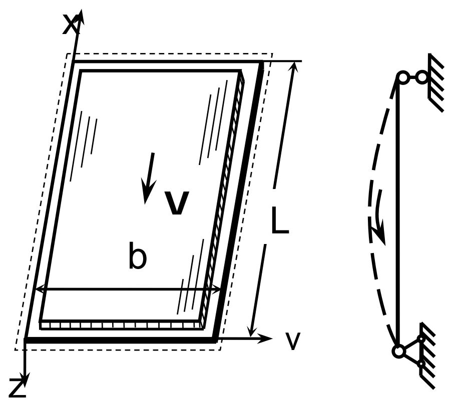

Figure 1: The mechanical model of the plate with the movable inertia loading.

The partial differential equation (9) describes free oscillations of the plate in relation to its quasi-static state. The partial solutions of the equation (9) with respective boundary conditions (10) will be found in the following form [4]

$$w(x,y,t)=\varphi(x,y)\cos\omega t+\psi(x,y)\sin\omega t\ . \tag{12}$$

After substitution of expression (12) to the differential equation (9) and introducing of complex functions of real arguments

$$\Phi(x,y)=\varphi(x,y)+i\psi(x,y) \tag{13}$$

the solution of the differential equation (9) after separation of variable x, y and t could be reduced to the differential equation of function Φ (x,y)

$$D\frac{\partial^4\Phi}{\partial x^4}+2D\frac{\partial^4\Phi}{\partial x^2\partial y^2}+D\frac{\partial^4\Phi}{\partial y^4}+\frac{q_1v^2}{g}\frac{\partial^2\Phi}{\partial x^2}-2i\omega\frac{q_1v}{g}\frac{\partial\Phi}{\partial x}-\frac{q_0+q_1}{g}\omega^2\Phi=0\ , \tag{14}$$

The solution of this equation could be obtained with accounting of boundary conditions (10) in the following form

$$\Phi(x,y)=\sum_m F_m(x)\sin\frac{m\pi}{b}y\ . \tag{15}$$

After substitution of expression (15) to the equation (14) we could obtain a fourth order ordinary differential equation with constant coefficients for $F_m(x)$ function, to solve that equation we have found the roots of algebraic equation of fourth power with complex coefficient. Boundary conditions (10) for function $F_m(x)$ will have following representation

$$F_m(0)=0,\ F_m(1)=0,\ F_m^{II}(0)=0,\ F_m^{II}(1)=0 \tag{16}$$

After determining of the roots of the characteristic equation and building-up $F_m(x)$ we have to match this function to the boundary conditions (16). Matching of $F_m(x)$ function to boundary conditions leads to determining of eigenvalues for when the fourth-order determinant with complex elements is equal to 0 [4,5].

The general solution of equation (14) with (16) obtains in the form

$$\begin{aligned}\Phi(x,y)=&\sum_{m,n}\left\{\left[T_n(x)+B_{2n}U_n(x)\right]\cos\delta_n x+\left[B_{1n}S_n(x)+B_{2n}\mathrm{sh}\beta_n x\right]\sin\delta_n x\right\}\sin\frac{m\pi}{b}y+\\ +i&\sum_{m,n}\left\{\left[V_n(x)+B_{2n}S_n(x)\right]\sin\delta_n x+\left[B_{2n}\mathrm{sh}\beta_n x-B_{1n}U_n(x)\right]\cos\delta_n x\right\}\sin\frac{m\pi}{b}y,\end{aligned} \tag{17}$$

where the expressions for the forms of their own and the accompanying vibrations for the rectangular plate should have following form:

$$\varphi_{nm}(x)=\left[T_n(x)+B_{2n}U_n(x)\right]\cos\delta_n x+\left[B_{1n}S_n(x)+B_{2n}\mathrm{sh}\beta_n x\right]\sin\delta_n x\ ,$$

$$\psi_{nm}(x)=\left[V_n(x)+B_{2n}S_n(x)\right]\sin\delta_n x+\left[B_{2n}\mathrm{sh}\beta_n x-B_{1n}U_n(x)\right]\cos\delta_n x$$

where $T_n(x)$, $U_n(x)$, $S_n(x)$, $V_n(x)$ – modified Krylov's functions [5]. Finally, the general solution of mathematical model (9)-(11) will have the following representation

$$w(x,y,t)=\sum_{n,m}a_{n,m}\left[\mathrm{Re}\left(\Phi_{n,m}(x,y)\right)\cos\left(\omega_{n,m}t+\alpha_{n,m}\right)+\right.$$
$$\left.+\mathrm{Im}\left(\Phi_{n,m}(x,y)\right)\sin\left(\omega_{n,m}t+\alpha_{n,m}\right)\right], \quad (18)$$

where arbitrary constants $a_{n,m}$ и $\alpha_{n,m}$ could be determined from initial conditions (11).

The procedure for building of the solution of the partial differential equation (9) with corresponding boundary conditions requires of some operations, that can't be done in analytical form without of using of information technologies. It is related to estimation of roots of algebraic equations of fourth order with complex coefficients, evaluation of determinant with complex elements, determination of parameters where it equals to 0 and etc [4]. In the same time to build and study of oscillations and stability by using of methods of two-wave representation it is needed to know the initial and basic values of some parameters that have to be specified in a further. Namely from this view point it is interesting to consider the stability of the rectangular plate compressed in one direction by normal stresses, located in vertical plane. For the first rough approximation, neglecting by Coriolis' force and considering the plate situated in a vertical plane in the neutral indifferent state we could get following equation of motion for the middle surface of the plate.

$$D\frac{\partial^4 w}{\partial x^4}+2D\frac{\partial^4 w}{\partial x^2\partial y^2}+D\frac{\partial^4 w}{\partial y^4}+\frac{q_1 v_{cr}^2}{g}\frac{\partial^2 w}{\partial x^2}=0\ . \quad (19)$$

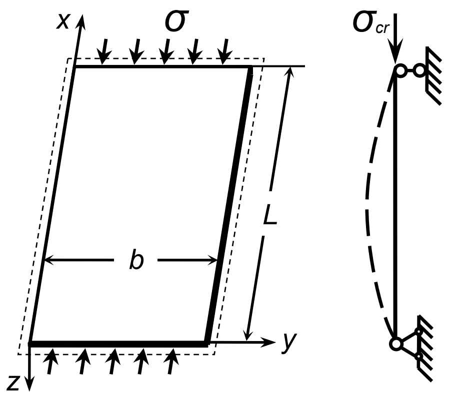

Figure 2: The mechanical model of the plate compressed by normal stresses.

It is simply to show, that the problem of determining of critical compressing normal stress for such plates (Fig.2) could be reduced to solving of the following differential equations [1,2]

$$D\frac{\partial^4 w}{\partial x^4}+2D\frac{\partial^4 w}{\partial x^2\partial y^2}+D\frac{\partial^4 w}{\partial y^4}+\sigma_{cr}h\frac{\partial^2 w}{\partial x^2}=0\ . \quad (20)$$

By comparing together of the equations (19) and (20) we could see clear that they are formally matched and it allows to make conclusion that the action of the movable inertia loading applied to the plate could be reduced to the acting of compressing normal stresses, and will have the same

$$\frac{q_1 V^2{}_{cr}}{g}=\sigma_{cr}\cdot h\ . \quad (21)$$

We could get the general expression for the approximate value of the critical speed of the loading stream motion

$$V_{cr}=\sqrt{\frac{\sigma_{cr}\cdot hg}{q_1}}\ . \quad (22)$$

Following by [1] we could obtain the expressions for determining of the critical normal stress for the plate with pinned sides b and free on each other sides as (23)

$$\sigma_{cr} = k_0 \frac{\pi^2 E}{12\left(1-\mu^2\right)}\left(\frac{h}{b}\right)^2 \tag{23}$$

and, respectively, for the critical speed

$$V_{cr} = \frac{\pi h}{b}\sqrt{k_0 \frac{Eh}{12\left(1-\mu^2\right)}\frac{g}{q_1}}. \tag{24}$$

The coefficient k_0 could be determined [1] from the table 1 in dependency from ratio between sides $\gamma = L/b$.

Table 1: The values of coefficient k_0 in dependency from ratio between lengths of the plate sides.

γ	0,5	1	1,5	2	2,5	3,0
k_0	3,83	1,039	0,47	0,27	0,18	0,13

In the case of simply supported edges we could get following expression for the critical value of compressing normal stress

$$\sigma_{cr} = k_1 \frac{\pi^2 E}{12\left(1-\mu^2\right)}\left(\frac{h}{b}\right)^2. \tag{25}$$

Values of coefficient k_1 in dependency from ratio between lengths of sides from respective table [1]. In general case the critical value of the normal stress could be determined as following

$$\sigma_{cr} = k_i \frac{\pi^2 E}{12\left(1-\mu^2\right)}\left(\frac{h}{b}\right)^2, \tag{26}$$

where coefficient k_i we could find from the respective tables by the boundary conditions on the plate edges and ratio its dimensions [1,2]

Analogies between the mathematical models of the elastic objects under movable load and of the elastic objects of varying length

Similar, it is simply to ascertain that the mathematical models of dynamics of the elastic object under movable inertia loading and the elastic object of time-varying length. The problem of lateral vibrations of the strengthen string by force and distributed weight q_1 under distributed loading moving with velocity V could be reduced to solving of differential equation [4,5]

$$\frac{\partial^2 u}{\partial t^2} + 2a\frac{\partial^2 u}{\partial t \partial x} - b^2\frac{\partial^2 u}{\partial x^2} = 0, \tag{27}$$

where $a = \frac{Vq_2}{q_1+q_2}$; $b^2 = \frac{Ng - V^2 q_2}{q_1+q_2}$,

q_1 - weight per unit length of the string;
q_2 - intensity of the movable load;
N - axial tension force;
V - speed of movable load.

The problem of first general problem of the dynamic of elastic objects with time-varying length based on differential equation [3]

$$\frac{q}{g}.\frac{\partial^2 u}{\partial t^2} - EA\frac{\partial^2 u}{\partial x^2} = q\left(1 \pm \frac{\ddot{l}}{g}\right) \tag{28}$$

in the time-varying ranges of integration $l(t) \le x \le l_0$,

where

u - extension of the element of the object;
q − is weight of the unit of the length of the object;
A − area of the cross-section;
E – Young’s module of the elasticity of the object;

g − acceleration of gravity;
l = l (t) − is the time − varying length.

After the variable substitution

$$y = \frac{l_0(x-1)}{l_0 - 1}$$

the equation (28) will be converted to

$$\frac{\partial^2 u}{\partial t^2} + 2a(y)\frac{\partial^2 u}{\partial y \partial t} - b^2(y)\frac{\partial^2 u}{\partial y^2} = 0 , \tag{29}$$

where in the permanent ranges $0 \le y \le l_0$ of integration

$$a(y) = \frac{l(l_0 - y)}{l_0 - 1}; \quad b^2(y) = \frac{\frac{g}{q}EAl_0^2 - \dot{l}^2(l_0 - y)^2}{(l_0 - l)^2} . \tag{30}$$

The differential equations (27) and (29), that describes the elastic object motion with time-varying length and objects with movable inertia loading by its representation are identical, that accentuates analogue of their mathematical models

Analysis of mathematical models and results

In contradiction to the classical schema of the variable separation method for the equation (27) we will be getting the solution in the following form

$$u(x,t) = \varphi(x)\cos\omega t + \psi(x)\sin\omega t . \tag{31}$$

Finally, we will get the general solution of the equation (27) in the form of superposition of two groups of standing waves

$$u(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{l_0}\cos\frac{n\pi a x}{l_0\sqrt{a^2+b^2}}\cos(\omega_n t + a_n) - \sum_{n=1}^{\infty} \sin\frac{n\pi x}{l_0}\sin\frac{n\pi a x}{l_0\sqrt{a^2+b^2}}\sin(\omega_n t + \alpha_n) \tag{32}$$

If the velocity of movable loading V is 0, then the solution (32) of the equation (27) goes over to known classical one.

$$u(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{l_0}\cos(\omega_n t + \alpha_n) \tag{33}$$

Oscillations from the first group, those modes for V-> 0 change over to classical oscillation modes of the object called natural oscillations, and oscillations form the second group are called accompanying oscillations of the object.

Analogously, oscillations of the object with time-varying length could be represented in the form of two groups of oscillations natural and accompanying oscillations, and their partial solutions we will get in the form (31). Thereafter the equations for oscillation modes will have varying coefficients.

$$\Phi''(y) + 2i\omega\frac{a(y)}{b^2(y)}\Phi'(y) + \frac{\omega^2}{b^2(y)}\Phi(y) = 0. \tag{34}$$

By substitute $\Phi(y) = \psi(y)\exp\left\{-\int_0^y \frac{i\omega a(y)}{b^2(y)}dy\right\}$

the equations (34) could be reduced to the form

$$\psi''(y) + \frac{\omega^2(a^2+b^2)}{b^4}\psi(y) = 0 , \tag{35}$$

where a and b could be obtained from (30). The modes of the eigenvibrations and accompanying oscillations of the elastic object with time-varying length will be getting as following

$$\varphi_n(y) = \psi^*_n(y)\cos\int_0^y \frac{a\omega_n}{b^2}dy\ ,\ \ \psi_n(y) = -\psi^*_n(y)\sin\int_0^y \frac{a\omega_n}{b^2}dy\ . \tag{36}$$

where $\psi^*_n(y)$ − solution of the equation (35), that obeys to the boundary conditions of the problem.

When the velocity of length varying less than the speed of elastic wave propagation, the equation (35) will not have singularities $0 \le y \le l_0$ and its solution could be obtained in the form of series [3]. The solution of the equation (35) build by the asymptotic method of Bogolyubov−Mitropolsky could be written as follows

$$\psi^*(y) = \sin\left[ky - k\frac{\dot{l}^2}{c}.\frac{l_0}{3}\left(\frac{l_0 - y}{l_0}\right)^3 + k\frac{\dot{l}^4}{c^4}.\frac{l_0}{10}\left(\frac{l_0 - y}{l_0}\right)^5 + \alpha\right], \tag{37}$$

where

$$k = \frac{\omega_n(l_0 - l)}{cl_0},\ \ c = \sqrt{\frac{g}{q}EA}\ .$$

Conclusions

The given analogy of mathematical models of some problems of the dynamics of elastic objects shows the affinity of these problems in their mathematical formulation. Naturally, this kinship also affects the nature of the movements of the objects being studied. Representatively that it is carried out in the form of two groups of oscillations, namely, own and accompanying, having the same frequencies, different forms and lagging. Accompanying oscillations are nontrivial in presence of moveable inertia loading or for the objects with time-varying length and significantly depend from the speed ratio between moving and stationary masses of the system.

Also, the own forms of oscillation significantly changeable, and in absence of moving inertia loads, they are convertible into their classical forms of oscillation. As for objects with time-varying length, these problems lie in outside of the scope classical problems of mathematical physics due to that the eigenfrequencies and eigenforms become time-dependent functions. This non-classical section of the mathematical physics is waiting for its development, new researches and generalizations.

In general, the problems of the dynamics of elastic systems under moveable inertia loadings, by its formulation, contain an independent section of the mathematical physics, structural mechanics of elastic systems due to specific of their formulations, methods of research, main quantitative and qualitative results, as well as importance for the practice development and exploitation of engineering structures. The essential feature of such systems is the presence of a load, which is represented in the mathematical model as an inertia operator on one of its form.

At the stage of development and exploitation of engineering structures under movable mass loading it is necessary do not forget, that the critical velocity of their moving at what loss of stability is possible can be quite small, achievable in practice, especially in cases of acting compressing forces on structural elements, close to the critical values according to Euler.

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