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DURABILITY OF THERMOVISCOELASTIC BODIES UNDER LONG-TERM CYCLIC LOADING

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An energy failure criterion for thermoviscoelastic structural members subject to abrasive wear and dissipative heating is proposed. The criterion is used to develop an algorithm to calculate the durability of such members under long-term cyclic loading. A simple formula of the number of cycles to failure is obtained. Experimental results on destruction of a rubber liner are given. Good agreement between the theoretical and experimental results is observed.

Keywords: viscoelastic structure, cyclic loading, failure criterion, rubber liner

Introduction. Rubber lining (rubbering) for various structural elements is widely used in metallurgy, mechanical engineering, processing, mining, beneficiation, chemistry, etc. It prevents the metal elements of equipment from failure, extends their service life by reducing wear, protects against noise, corrosion, shock, and high temperature, and significantly reduces the load on these elements. Cyclic stressing is one of the principal types of the lining loading. Rubber lining has viscoelastic properties and is heated during cyclic loading due to the hysteresis losses. Therefore, calculating the durability of a rubber lining under loads, taking into account the indicated material behavior, is a crucial problem in solid mechanics.

This article proposes an energy failure criterion for viscoelastic structural elements, including a rubber lining, under long-term cyclic loading. This criterion was used to develop an algorithm for calculating the durability of such elements under long-term cyclic loading. A simple formula is obtained to determine the number of cycles to failure. The experimental results on destruction of a rubber lining are presented, indicating a satisfactory agreement between the theoretical and experimental data.

1. Generalized Algorithm. The structural-synergetic model of lining failure can be represented as follows: the destruction of the lining on the surface occurs in two ways: (i) abrasive wear, i.e., removal of rubber aggregates by harder counter bodies and (ii) micro- and macro fatigue failure of rubber. The combination of these two mechanisms leads to abrasion-fatigue failure of rubber. The damage rate is generally determined by the fatigue damage of the rubber.

The lining durability algorithm is based on an integrated approach to lining design, combining analytical calculations and experimental data on fatigue, thermo-mechanical, rheological characteristics of rubber, type of wear and abrasion of the rubber surface and its failure model. It includes the solution of the following problems:

– calculation of the lining stress–strain state, taking into account the viscoelastic effects in rubber;

– solving the heat-transfer equation with an internal heat source to determine the temperature of dissipative heating in a rubber body;

– use of the criterion equations relating the failure parameters (abrasive-fatigue wear and deterioration of the material with time) and the time to failure according to the criteria adopted in engineering.

Let us use this algorithm the design a single lining plate.

The stress–strain state and the dissipative heating temperature of viscoelastic structural elements are calculated in [1, 2, 6–8].

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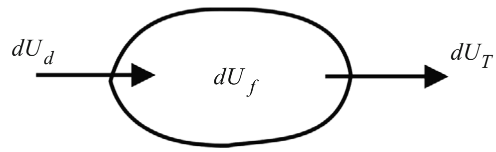

Fig. 1

Double-Criteria Durability Equation of a Rubber Lining (RL). For open thermodynamic systems that exchange an energy with the environment, the dissipation energy dU_d during dt can be resolved into components [3]:

$$dU_d = dU_T + dU_f,$$

where dU_T is the thermal energy flux into the environment (Fig. 1); dU_f is the energy of the irreversible processes within the system, i.e., destruction of rubber ; for all real processes the work of destruction is given by

$$dU_f > 0.$$

For abrasive-fatigue mechanism of failure of RL, the total failure energy ΔU_f is expressed as

$$\Delta U_f = \Delta U_y + \Delta U_w,$$

where ΔU_y is the energy of destruction of rubber bulk; ΔU_w is the energy of destruction of the surface layer of rubber due to abrasive wear.

We will consider the deformable lining plate as a thermodynamic system. It is known that the state of any thermodynamic system is most fully characterized by its internal energy. Therefore, we suppose that there is a one-to-one correspondence between the degree of damage to the system and its internal energy. Based on this assumption, we will derive the criterion failure equation.

Let us write the first law of thermodynamics for the case where the mechanical forces and some non-mechanical forces such as radiation, act on the sample:

$$\dot{U} = \sigma_{ij}\dot{\varepsilon}_{ij} + \dot{\chi} + \dot{\xi},$$

where U is the internal energy of the system; χ is the non-mechanical energy; ξ is the abrasive wear energy (hereafter, the time derivative is marked by a overdot).

These forces increase the internal energy of the system. However, every system tends to pass into the state with minimal energy. Therefore, increased internal energy is consumed within the system. According to the first law of thermodynamics, the work done inside the system is aimed at changing the internal structure of the system and heat generation, i.e.,

$$\sigma_{ij}\dot{\varepsilon}_{ij} + \dot{\xi} + \dot{\chi} = \dot{U}_p + \dot{q},$$

where U_f is the part of the internal energy that is used to change the structure of the system, i.e., to cause destruction; q is the part of the internal energy dissipated as heat.

After time t^* the energy balance will be as follows:

$$\Delta U_f^* = \int_0^{t^*} (\sigma_{ij}\dot{\varepsilon}_{ij} - \dot{q} + \dot{\xi} + \dot{\chi})dt. \quad (1)$$

Knowing the external forces acting on the system and time t^*, from equation (1), we can determine the value of ΔU_f^*, and, alternatively, knowing ΔU_f^* and external forces, it is possible to find the time to failure t. Thus, Eq. (1) is a long-term strength criterion and allows us to determine the time to failure of a characteristic volume of the body under the known deformation conditions and for the experimentally found constant ΔU_f^*.

The equation for local durability of the rubber plate (N^* is the number of cycles to failure) has the form:

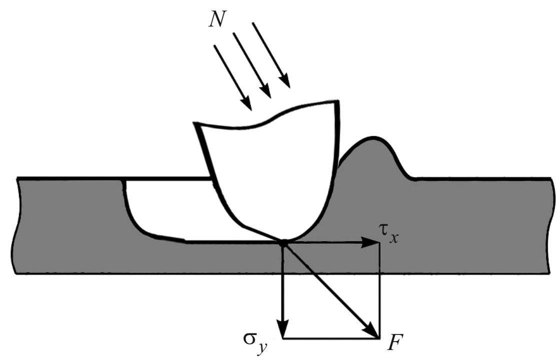

Fig. 2

$$N^* = \frac{\eta_l \delta_m \Delta U_f^*}{0.5|E^*|\varepsilon^2 \psi(1-\eta_T)}, \qquad f(x, y, z), \tag{2}$$

where δ_m is the coefficient of asymmetry of the mill charge; $|E^*|$ is the absolute value of the complex elasticity modulus; η_l is the coefficient that defines the rubber lining profile; ε is the relative strain of the lining plates; η_T is the coefficient that shows how much of the energy dissipated in the rubber is spent for heat generation; the function $f(x, y, z)$ defines the stress and strain distribution.

Lining plates under the loading are characterized by a stationary dissipative self-heating temperature field. Therefore, in the first approximation, the parameter η_T can be taken independent of the loading conditions and the temperature of the environment.

Under these assumptions, the criterion equation (2) becomes simpler:

$$t^* = N^* = \frac{\eta_l \delta_m \Delta U_f^*}{0.5|E^*|\varepsilon^2 \psi \eta_f}, \tag{3}$$

where $\eta_f = 1-\eta_T$ is a coefficient that defines the part of the energy used directly for destruction of the rubber.

Taking into account

$$\eta_f = \frac{\Delta U_f^*}{\Delta U_d^*}$$

gives

$$N^* = \frac{\eta_l \delta_m \Delta U_d^*}{0.5|E^*|\varepsilon^2 \psi}, \tag{4}$$

where ΔU_d^* is the critical (allowable) density of the energy dissipated in a lining plate under the loading.

Note that the time to local failure of t^* the lining plate is identified with the destruction of the central region of the rubber bukl.

For 541933-1 rubber, the allowable fatigue failure energy density found in direct experimentals is $\Delta U_d^* = 1.28 \cdot 10^{10}$ J/m^3.

2. Phenomenological Model of Abrasive Wear for RL. When the mill charge and the RL interact, sharp and solid counter bodies (balls and particles of the processed material) are pressed into the surface layer of rubber at a certain angle and remain there for some time. The nature of the interaction of the abrasive particles with the surface layer of the rubber lining is shown in Fig. 2, where N is the external force; σ_y is the normal stress; τ_x is the shear stress; F is the resultant force.

In front of the sharp protrusion of the counterbody, the rubber is in a compressed state, and behind it experiences large tensile strains, which leads to tearing of the rubber and the formation of strips in the direction of sliding. It is the presence of longitudinal stripes on the abrasion surface that is a manifestation of abrasive wear. At the same time, large contact stresses lead to the destruction of local volumes of rubber, to cuts (tear) and to mass transfer of rubber particles, i.e., to the separation of particles and their removal from the drum by the coolant. A complex stress state arises in a local volume of rubber, which, in the simplest case, can be reduced to the normal, σ_y, and tangential, τ_x, stresses (see Fig. 2), according to the Hertz–Dinnik model. The abrasive wear intensity is determined by the load, hardness and sharpness of the counterbody protrusions, the angle of attack, and the mechanical characteristics of the rubber. Thus, the mechanism of abrasive wear is quite complicated; account must also be taken of the nonlinearity and stochastic behavior of this process and the fact that the rubber in the surface layer has a higher degree of damage than in the volume [3].

Thus, the abrasive wear of rubber can be represented as mechanical detachment of certain particles (aggregates) of material, and the wear resistance in this case, of course, is due to the strength properties of rubber. Wear resistance also depends on the temperature and, generally, follows the concept of temperature–time superposition, similar to rubber strength. This fact is important because it indicates the common viscoelastic nature of the destruction of rubbers during mechanical breaks and abrasion, which allows us to use the same physical models, algorithms, and failure criteria to describe the process.

Determination of the Failure Energy during Abrasive Wear of Rubbers. As already mentioned in [4], the failure of rubber lining of the drum mills follows the abrasive-fatigue mechanism. Consider the abrasive component of this complex multi-parameter process. It is known that abrasive wear is based on the following components: wear caused by the breaking of internal cohesive bonds of the material; adhesion due to the molecular forces; deformation caused mainly by the dissipative forces.

The modern experimental capabilities make it possible to isolate, mainly, the adhesive component of wear, which is caused by the breaking of "molecular aggregates" (the term used by Shallamakh, more suitable for the process of macrocracking [5], Grosh [6], and others). The term "aggregate" is used below for abrasive wear of a rubber lining.

We will make the following important assumptions that give a general idea of the mechanism of rubber destruction due to abrasive wear and stay within the accepted phenomenological model:

– the matrix and the counterbody (i.e., the rubber lining and the charge) undergo uniform relative slip with speed V;

– the temperature T in the contact zone does not exceed the permissible temperature $[T]$ for the type of rubber under consideration: $T < [T]$;

– the process under study is compatible with the principle of equivalence of speed and temperature, i.e., $A(V, T)$ that depends on the speed and temperature complies with the principle of temperature–time superposition (the so-called Williams–Landel–Ferry (WLF) equation); this principle is described in [1, 2] for the deformation characteristics of rubber, and in [4] for the wear characteristics;

– a rubber aggregate is separated from the matrix, i.e., the bond is broken, when the energy accumulated by the aggregate during wear reaches a certain critical value U_0; in studying single acts of wear (scratching with a needle) of filled rubbers, Shallamah, Grosh and, later, Hatfield showed (including experimentally) that the energy absorbed by the "molecular aggregate" during wear is of the order of magnitude that the energy barrier between free and bound aggregate states; such a concept of abrasive wear destruction is justified by these authors;

– the relaxation function of aggregates is known and determined by the properties of the material for the rubber under study. Moreover, knowing the relaxation function and the law of displacement of two materials in contact (uniform relative sliding at a given speed) and using the Boltzmann integral, we can obtain the bonding force equation for an elementary aggregate to calculate the friction (wear) force as the average of the bonding forces.

Using the known dynamic limit of one bond (its value can be found experimentally, based on the value of friction at speeds close to the speed at zero slip), the obtained wear force, and the energy failure criterion, we can determine the critical failure energy of the rubber during abrasive wear.

Energy Failure Criteria for Rubber Subject to Abrasive Wear. When a rubber aggregate separates from the matrix, it is assumed that the rubber relaxation function $r(t)$ is known (i.e., we know the mechanical parameters of rubber) and there is a uniform relative motion between the charge and the lining, with a low constant speed V in most cases. Using the Boltzmann integral, we can get the equation of the bonding force for an elementary aggregate of rubber and then determine the friction force (wear) as the average of the bonding force.

Suppose n is total number of aggregates subject to the bonding forces near the contact surface; n_0 and n_1 are the number of aggregates, respectively, in a bound and free (i.e., after separation) state; t_0 and t_1 are the times during which the aggregate is in a bound and free states, respectively; these quantities are related by the statistical relations

$$\frac{n_0}{t_0} = \frac{n_1}{t_1} = \frac{n}{t_0 + t_1}. \tag{5}$$

The assumption that the time during which the aggregate is in a free state is proportional to the relaxation time τ of the aggregate,

$$t_1 = a\tau,$$

where a is a constant, will be true if we assume that the time required for the aggregate to reach a known dynamic level is proportional to τ, and the displacement will be proportional to V.

Using the above assumptions and the Boltzmann integral, we determine the bonding force $f(t)$ as

$$f(t) = V\int_0^t r(t-t')dt'. \tag{6}$$

If we know the dynamic limit f_0 of one rubber aggregate, then it can be determined experimentally, from the friction at speeds close to the speed at zero slip (for example, when using lubricant) using the expression

$$f_0 = 2F(0)/n_0, \tag{7}$$

where $F(0)$ is the zero slip friction force.

Assuming that the bond between the aggregates of rubber disappears when the force reaches the value f_0, we can write Eq. (6) as follows:

$$f(t_0) = f_0. \tag{8}$$

The total friction force, as the average of the bonding forces of the aggregates in contact with the counterbody, is:

$$F = \frac{n_0}{t_0}\int_0^{t_0} f(t)dt. \tag{9}$$

Suppose that the rubber is characterized by the relaxation function

$$r(t) = E_0(1 + be^{-t/\tau}), \tag{10}$$

where E_0 is the elasticity modulus of rubber; τ is the relaxation time; b is some constant; t is the current time.

The elementary bonding force of each aggregate is determined in accordance with (6) for a given form of function (5) from the expression

$$f(t) = vtE_0 + \tau vbE_0 - vbE_0\tau e^{-t/\tau}. \tag{11}$$

Let

$$L = v\cdot\tau \quad \text{and} \quad \alpha = t/\tau, \tag{12}$$

where L is the relaxation length of molecule aggregate; τ is the relaxation time.

Then formulas (11) can be written as

$$f(t) = LE_0\left[\alpha + b(1 - e^{-\alpha})\right]. \tag{13}$$

The quantities τ and L are of the same order of magnitude with, respectively, the mean free time of rubber aggregates and their mean free path [4, 5].

Using notation (12), from conditions (8)

$$f_0 = f(t - t_0) = E_0 L\left[\alpha_0 + b(1 - e^{-\alpha_0})\right], \tag{14}$$

and the equation

$$L = \frac{f_0}{E_0}\left[\alpha_0 + b(1 - e^{-\alpha_0})\right]^{-1}, \tag{15}$$

we can determine t_0 (as well as a_0).

The friction force F (total force) is determined by averaging the bonding forces of rubber aggregates according to the formula

$$F = \frac{n_0}{t_0}\int_0^{t_0} f(t)dt. \tag{16}$$

After integration, the general wear equation (16) takes the form

$$F = \frac{n_0 E_0 L}{\alpha_0}\left[\frac{\alpha_0^2}{2} + b(\alpha_0 + e^{-\alpha_0} - 1)\right] \tag{17}$$

or taking into account (5),

$$F = \frac{nE_0 L}{\alpha_0 + a}\left[\frac{\alpha_0^2}{2} + b(\alpha_0 + e^{-\alpha_0} - 1)\right]. \tag{18}$$

When studying the change in the adhesive component of friction as a function of the sliding speed $F(L)$, it is sufficient to eliminate a_0 by simultaneously solving Eqs. (15) and (18) and to analyze the obtained results.

In view of the foregoing, we may state that the energy failure criterion for rubber, which postulates that the breaking of the bond between the aggregate and the matrix occurs when the energy accumulated by the aggregate during the wear of rubber reaches a certain critical value U_0, allows us to determine the value of U_0 according to the equation

$$U_0 = V\int_0^{t_0} f(t)dt \tag{19}$$

or, taking into account (9) and (12),

$$U_0 = Vt_0 F / n_0. \tag{20}$$

According to (5),

$$\frac{n_0}{t_0} = \frac{n}{t_0 + t_1} = \frac{n}{\alpha_0\tau + a\tau} = \frac{n}{\tau(\alpha_0 + a)}$$

formula (20) is transformed to

$$U_0 = \frac{L(\alpha_0 + a)}{n}F \tag{21}$$

or in terms of the force of friction:

$$F = \frac{nU_0}{L(\alpha_0 + a)}. \tag{22}$$

For rubber with relaxation function in the form (10), the expression for the parameter L can be derived using formulas (21) or (22) and the evaluated integral (18):

$$L = \sqrt{\frac{U_0 / E_0}{\frac{\alpha_0^2}{2} + b(\alpha_0 + e^{-\alpha_0} - 1)}}. \tag{23}$$

The change in the friction force F, as a function of the speed $F(L)$ is found by solving the system of equations

$$\begin{cases} L = \sqrt{\dfrac{U_0 / E_0}{\frac{\alpha_0^2}{2} + b\left(\alpha_0 + e^{-\alpha_0} - 1\right)}}, \\ F = \dfrac{nU_0}{L(\alpha_0 + a)}. \end{cases} \tag{24}$$

Thus, for the filled rubber used as protective lining for a drum of the ore-grinding mills, it is advisable to determine the abrasive fatigue failure energy by formula (20) with known relaxation curve and experimentally found wear parameters of model samples.

Experimental Studies. Such studies were performed in accordance with GOST 426-77 (Method for determining the resistance to abrasion in sliding). We used a MI-2 test bench and standard 20×20×8 mm samples of 541933-1 lining rubber, attached to a special holder frame and abraded with a grit cloth (according to GOST 344-74). For statistical data processing, at least nine tests were conducted. The results thus obtained were as follows: friction force $F = 16$ N, abrasion time $t = 150$ sec, abrasion speed $V = 0.285$ m/sec.

The number of wear particles $n = 60 \cdot 10^3$ (average over nine tests; the average mass of particles is 0.5 g; at the average particle diameter $d = 0.4$ mm the number of particles in one cubic meter is $n^* = 22 \cdot 10^9$ m^{-3}).

In this case, the failure energy for one rubber fragment (i.e., the energy of its separation from the matrix) according to Eq. (20) is

$$U_0 = \frac{FVt}{n} = \frac{16 \times 0.285 \times 150}{6.0 \times 10^3} = 114 \times 10^{-3} \text{ J.}$$

The density of abrasive wear failure energy, i.e., failure energy per unit volume of the material, is

$$\Delta U_w = U_0 n^* = 114 \times 10^{-3} \times 22 \times 10^9 = 0.25 \times 10^{10} \text{ J/m}^3.$$

The experimental studies show that even for the material from the same batch of rubber lining plates, the failure energy density is within $(0.22 - 0.28) \times 10^{10}$ J/m^3, which is understandable, given the stochastic nature of wear and technology factors.

3. Example of Calculating the Durability of Rubber Lining of Drum Grinding Mill.

Prerequisites for the Calculation. In addition to the general theoretical models, the wear curves for rubber lining based on many-years experimental observations, are also the prerequisite for calculation of the local durability t^*. Figure 3 shows such a wear curve for a Plita–Volna rubber lining installed in a 3.2×34.5 CDB mill.

The point F corresponded to the local durability of the RL, i.e., $t^* = 0.87 t_E^*$, where $t_E^* = 23100$ h is the operating time determined from the failure criteria; the point A corresponded to wear such that the residual RL thickness is $[\Delta h] = 40$ mm, i.e., during normal operation; the point B corresponded to wear such that the residual thickness of the RL is $[\Delta h] = 27$ mm, i.e., when the mill operates in pre-emergency mode; after that, the RL was dismantled. The time to failure of the RL for a particular mill was $t_E^* = 23100$ h; for the type of mills under consideration, the time to failure of the RL was (22–24)$\cdot 10^3$ h.

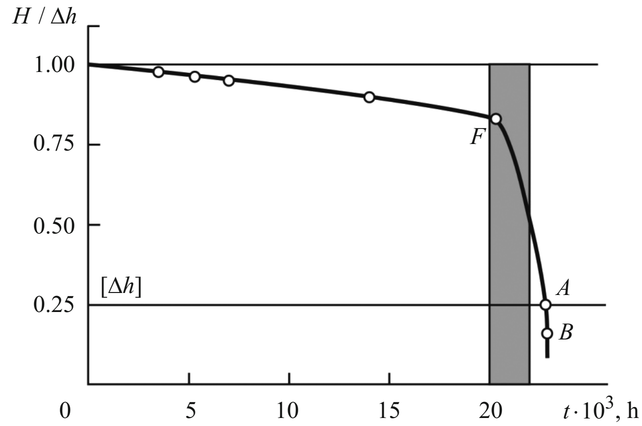

Fig. 3

The physical and mechanical characteristics of the rubber were determined using the model samples and the methods from [3].

The constants that characterize the design features of the RL were determined as follows:

the coefficient η_l that characterizes the profile of the RL is found as the ratio of the durability of trapezoidal plates with recesses; it was experimentally established that the durability of trapezoidal plates is 12 to 20% more than those of smooth plates; therefore, the coefficient was taken equal to $\eta_l = 1.12–1.20$;

the coefficient δ_m that characterizes the influence of load asymmetry along the mill length was adopted as the ratio of the RL durability for the moderate wear zone to the intensive wear zone (in the moderate wear zone, the RL durability was 10 to 12% higher than that in the intensive wear zone); therefore, the coefficient was adopted as $\delta_m = 1.12$. These coefficient were obtained during more than 15 years of operation of the CDB mill with rubber lining.

Input Data for Calculation.

1. CDB mill 3.2×4.5; charge: balls with a diameter of 40 mm; drum rotation speed $\omega = 19.8$ rpm ($\omega / 60 = 0.33$ cycle/sec).

2. Plita–Volna rubber lining; trapezoidal plates with recesses, maximum plate height h_l = 160 mm; experimental average deformation of the plates $\Delta_d = 6.4$ mm (compressive strain $\varepsilon = 0.04$); $\eta_l = 1.20$, $\delta_m = 1.12$.

3. Rubber 541933-1 with the following physical and mechanical characteristics: dynamic Young's modulus $E_d = 5.67 \cdot 10^6$ Pa; energy dissipation factor $\psi = 0.59$; factor $\eta_T = 0.75$; distribution function of the stress and deformation fields according to the method from [5] $f(x, y, z) = 1.23$.

4. Total experimental abrasive-fatigue wear failure energy of the RL is

$$\Delta U_f^* = \Delta U_y^* + \Delta U_w^* = (1.28 + 0.25) \times 10^{10} \text{ J/m}^3 = 1.53 \times 10^{10} \text{ J/m}^3.$$

Given these data, the number of cycles to local failure of the rubber lining can be determined as follows:

$$N^* = \frac{\delta_m \eta_l \Delta U_f^*}{0.5 E_d \varepsilon^2 \psi (1 - \eta_T) f(x, y, z)} = \frac{1.20 \times 1.12 \times 1.53 \times 10^{10}}{0.5 \times 5.67 \times 10^6 \times 0.04^2 \times 0.59 \times 0.25 \times 1.23} = 0.249 \times 10^8 \text{ cycles}$$

or

$$t^* = \frac{N^*}{\omega} = \frac{0.249 \times 10^8}{0.33} = 0.754 \times 10^8 \text{ sec} = 20\,960 \text{ h}.$$

The experimentally found local durability of the RL is 20,300 h (see Fig. 3); the operational durability of RL in the normal conditions is 23,100 h. As we can see, the agreement between the calculated and experimental data is quite satisfactory. Such agreement is not accidental: all the physical and mechanical characteristics of rubbers were determined experimentally; the parameters of the RL in a real mill were constantly monitored, recording the experimental wear curve; the calculation was carried out for the zone of moderate wear.

When using the above algorithm in the absence of reliable parameters (for example, when predicting the durability of the RL from a new rubber grade or new designs of lining plates), the error will certainly be slightly larger.

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