The equation of the modulator's passing cross-section area in the pulsation device with the vibrating rotor

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Abstract. The article provides an analysis of the existing analytical models of the change in the cross-sectional area of the modulator in rotary-pulsation devices. The peculiarities of the application of such functions for the pulsation devices with the vibrating rotor are determined, taking into account the predominant mechanism of dispersion of emulsions. Using classical hydraulic and mathematical dependencies, in order to fulfill the requirement of creating harmonic pulsations of the emulsion in the modulator holes for synchronization with the axial oscillations of the rotor and the equality of the conditions of pulsations of the speed of the emulsion in each channel of the stator and rotor, the equation of the change in the cross-sectional area of the modulator in the pulsation devices with the vibrating rotor is determined in the form of a continuous function. This greatly simplifies the mathematical description of the function, and the subsequent analytical model of the movement of the emulsion in devices of this type. The resulting equation is necessary in determining such characteristics as the instantaneous speed and acceleration of the emulsion and the diameter of the dispersed phase after treatment. Keywords: Rotary-pulsation device, pulsation device with the vibrating rotor, disperser, homogenizer, modulator area

1 Introduction

Dispersion is widely used in the production of butter, margarine, mayonnaise, creams, products with biologically active additives, etc [1, 2]. A particularly energy-consuming process is the dispersion of the fat phase of milk, the so-called homogenization of milk [3, 4]. Considering the obvious relevance of the problem of reducing the energy consumption of the homogenization process in the dairy industry [5, 6], a wide range of homogenization devices, such as high-pressure (valve) [7, 8] and ultra-high-pressure [9, 10], pulsation [11], vacuum [11], microfluidizers [12, 13], micromixers [14], jet and stream [17, 18], ultrasonic [19, 20], rotary [21], etc., have been developed. However, none of them combines a high degree of destruction of milk fat globules (as, for example, in valve devices) with low energy consumption [22, 23]. Rotor-pulsation homogenizers, the principle of operation of

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which is based on discrete-pulse input of energy, are approaching such "ideal" devices [24, 25]. But a more effective type of these devices, the rotor of which performs oscillating movements along the axis of rotation along with the rotational movement, are the so-called pulsation devices with the vibrating rotor (PD with VR) [26]. The quality of homogenization during milk processing in such machines is no less than in valve machines, and energy consumption is 4-5 times lower [27, 28]. To introduce such devices into production, it is necessary to develop a methodology for calculating structural and operational parameters.

During previous studies, it had been determined that in PD with VR, the destruction of fat globules mainly occurs according to the Rayleigh-Taylor instability mechanism, where the diameter of the dispersion particle depends on the acceleration of the liquid flow during the movement of the milk emulsion through the channels of the PD interrupter (rotor and stator holes) [25, 26] . The acceleration of the milk flow causes a difference in the speed (sliding) of the fat globule relative to the milk plasma, which leads to the destruction of the fat particle [29, 30]. To determine the average size of the fat globule after homogenization, it is necessary to calculate the value of the average acceleration of the milk emulsion. For this, it is necessary to determine the equation of the change in the cross-sectional area of the PD modulator with VR.

2 Theoretical Background

In rotary-pulsation devices (RPD), the law of change of the cross-sectional area of the modulator with time ***S(t)*** mainly determines the magnitude of pressure pulses and, as a result, the intensity of cavitation, which is considered the main reason for the destruction of the dispersed phase in rotary-pulsation dispersers. To create rarefaction in the RPD stator channels, which is necessary for the occurrence of cavitation, a sudden (quick) closing of the modulator is necessary [24, 31]. For this reason, the RPD holes have a rectangular shape and they try to make the gap $\boldsymbol{\delta}$ as small as possible. In addition, the most important period of time in the operation of the RPD is the closing of the modulator holes. Inaccuracies in the description of the function of the cross-sectional area of the modulator as a function of time ***S(t)*** in this interval lead to significant errors in determining the value of pressure pulses, and therefore the efficiency of the RPD as a whole [32].

For stator and rotor openings of a rectangular shape, the equation ***S(t)*** represents a trapezoid on the interval of the process of opening and closing the openings [24, 25]. Oval-shaped holes are reduced to rectangular with the help of appropriate expressions. Due to the small clearances required in RPD designed for maximum cavitation, the gap between the rotor and the stator is neglected. For PD with VR, there is no need to create a minimum gap, so its size cannot be underestimated [26].

In works [24], the function ***S(t)*** for the beginning of the hole closing process has a break in the derivative, which is unacceptable. The main reason for this is the representation of the function ***S(t)*** in the form of a piecewise function. The cycle of changing the area of the breaker is divided into 4 sections: opening of the modulator openings, fully open openings, closing of the openings, and fully closed openings – for each of which the function ***S(t)*** is determined. In [33], a model of the change in ***S(t)*** is proposed, which takes into account the flow through the modulator gaps not only in terms of width, but also in terms of the height of the modulator channel, but its use is limited to RPD with small $\boldsymbol{\delta}$ [34].

Thus, until now, sufficiently informative mathematical models of the change in the cross-sectional area of the RPD modulator have been developed. The main disadvantages of these models are the cumbersomeness and limitation - the use of equations for calculating the RPD only with large or only with small gaps between the rotor and the stator.

3 Research Methods

During the theoretical studies, the classical dependences of hydraulics and mechanics, the theory of discrete-impulse energy input, oscillations of conservative linear systems were used to study hydrodynamics in a virtual environment [35, 36]. To find the optimal process parameters, the two-parameter graphic method and analytical methods of local optimization were used [37]. Research was conducted according to the following plan:

– development of the calculation scheme of the rotor-pulsation devices;

– determination of kinematic and geometric requirements for the RPD to fulfill the condition of equality of emulsion pulsations;

– analytical-geometric transformations;

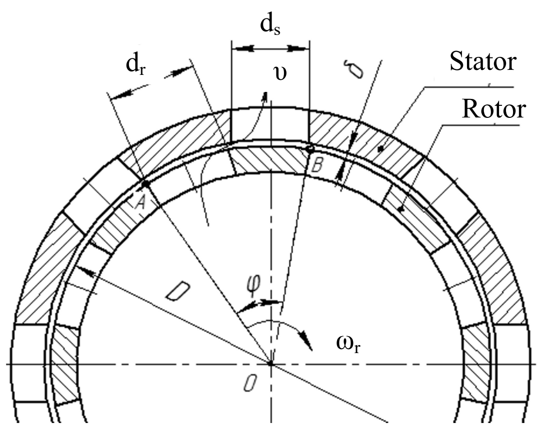
– obtaining and analyzing the equations of the area change of the RPD modulator;

– analysis of the resulting equation.

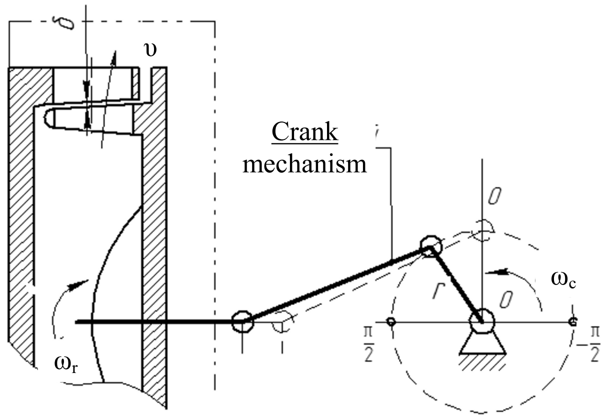
4 Results and Discussion

For a classic RPD, in order to obtain a high-quality emulsion (high degree of dispersity), it is necessary to fulfill the requirement of creating a minimum gap between the rotor and the stator. The size of such a gap should be 0.1-0.01 mm [24]. The fulfillment of such a requirement in engineering practice leads to a significant increase in the cost of industrial samples of RPD. For PD with VR, the determining factor of homogenization is the acceleration of the movement of the emulsion in the holes of the modulator, so there is no need to create minimal gaps $\boldsymbol{\delta}$ [26]. The second advantage of PD with VR compared to classic RPD is in the shape of the modulator holes. For RPD, for the purpose of creating a sudden (quick) closing-opening modulator, it is necessary to make holes of a rectangular shape. This is difficult for the cylindrical surface of the modulator. For PD with VR, it is possible to use round modulator holes, which is technologically simpler. In addition, there is no need to describe the function ***S(t)*** absolutely precisely when closing the modulator holes. It is more important to describe the dependence ***S(t)*** as a continuous function in order to avoid unnecessary cumbersomeness and the possibility of obtaining dependencies that are convenient for use in the calculation of PD from VR.

The calculation scheme of the pulsation devices with the vibrating rotor is presented in fig. 1.



a)



b)

Fig. 1: Scheme of rotational and oscillating movements of the PD rotor: a) rotational movement of the rotor, b) oscillating movement along the rotor axis. υ – emulsion speed; r – crank radius; d_s – the diameter of the stator holes; d_r – the diameter of the rotor holes; ω_c – crank speed; ω_r – speed of rotation of the rotor; φ – rotation angle of the rotor.

In order to fulfill the requirement of creating harmonic pulsations of the emulsion in the holes of the PD for synchronization with the axial oscillations of the rotor and the equality of the conditions of pulsations of the speed of the emulsion ***υ*** in each channel of the stator and the rotor, it is necessary to fulfill the conditions of equality [26]:

– the number of rotor and stator holes $z_r = z_s = z$;

– diameters of the rotor and stator holes $d_r = d_s$.

We assume that the rotor rotates with a constant angular velocity ω_r, and the crank for driving axial movements of the rotor with a speed ω_c.

In the position of the rotor shown in fig. 1 a, the stator holes are completely closed by the gaps between the rotor holes. At the same time, the area of the interrupter PD with VR ***S*** is minimal and equal

$$S_{min} = \pi d_r \delta z \,, \tag{1}$$

where δ – the radial gap between the rotor and the stator, m.

When the rotor rotates at speed ω_r, at some point in time, the rotor holes will coincide with the stator holes. In this position, the area of the breaker reaches its maximum value

$$S_{max} = \frac{\pi d_r^{\,2}}{4} z \,. \tag{2}$$

When turning the rotor from point A to point B (see Fig. 1 a), the area will again take on a minimum value. With further rotation, the cycle of changing the area of the breaker is repeated.

The change in the area of the PD breaker holes with round holes is described with satisfactory accuracy by the equations (in dimensionless form, where by $S(t)max = 1$ and δ=0)

$$S(t) = \frac{1}{2} + \frac{sin(t\omega_r z - \frac{\pi}{2})}{2} \,. \tag{3}$$

Or, taking into account that $t = \varphi / \omega_p$, as a function of the angle of rotation, where at the instant of time $t = 0$ corresponds to the one in which the rotor holes are covered by the gaps between the stator holes (see Fig. 1).

$$S(\varphi) = \frac{1}{2} + \frac{sin(\varphi z - \frac{\pi}{2})}{2}, \tag{4}$$

where φ – the rotation angle of the rotor, rad.

The cycle time of changes in the cross-sectional area of the modulator t_c, s, is equal to

$$t_c = 2\pi / z\omega_r, \tag{5}$$

Taking into account the gap between the stator and the rotor δ as a function of the rotor rotation angle, dependence (4) takes the form (Fig. 2)

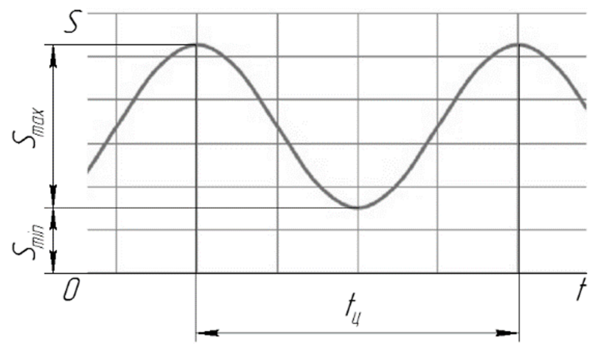


Fig. 2. The nature of the change in the PD modulator area with VR.

$$S(\varphi) = S_{max}\left(\frac{1}{2} + \frac{sin(\varphi z - \frac{\pi}{2})}{2}\right) + S_{min}, \tag{6}$$

where S_{max} – the maximum possible area of the circuit breaker PD, m^2;

S_{min} – the minimum possible area of the PD breaker (when the rotor holes are closed), m^2.

Taking into account (1) and (2), formula (6) takes the form:

– as a function of time

$$S(t) = \frac{\pi d_r^{\,2} z}{8}\left(1 + sin(t\omega_r z - \frac{\pi}{2})\right) + \pi d_r \delta z, \tag{7}$$

– as a function of the rotor rotation angle

$$S(\varphi) = \frac{\pi d_r^{\,2} z}{8}\left(1 + sin(\varphi z - \frac{\pi}{2})\right) + \pi d_r \delta z. \tag{8}$$

The number of rotor holes is related to the rotor diameter ***D***, m, and the diameter of the holes by a ratio that follows from obvious geometric transformations

$$d_r = \frac{\pi D}{2z}. \tag{9}$$

Taking into account the last expression, formulas (7) and (8) take the form:

– as a function of time

$$S(t)=\frac{\pi^2 D}{2}\left(\frac{\pi\ D}{16z}\left(1+sin(t\omega_r z-\frac{\pi}{2})\right)+\delta\right), \tag{10}$$

– as a function of the rotor rotation angle

$$S(\varphi)=\frac{\pi^2 D}{2}\left(\frac{\pi\ D}{16z}\left(1+sin(\varphi z-\frac{\pi}{2})\right)+\delta\right). \tag{11}$$

Let's analyze the obtained equations of the dependence of the cross-sectional area of the PD modulator with VR on time and the angle of rotation of the rotor. We see that the variable parameters of the equation are the diameter of the holes, the number of holes and the size of the gap between the rotor and the stator. The equation is a sinusoid that changes its values from the minimum (at t=0) to the maximum value (Fig. 3).

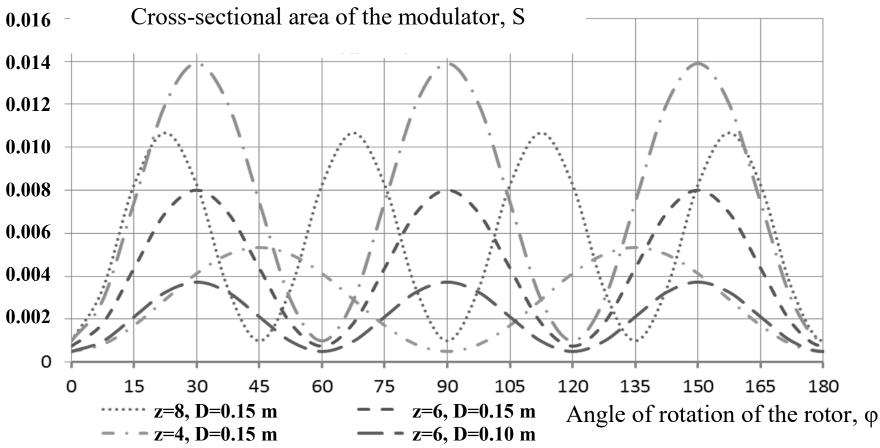


Fig. 3. Dependencies of the cross-sectional area of the modulator on the diameter of the rotor and the number of holes at =1mm.

The largest influence on the amplitude of oscillation of the modulator cross-section is exerted by the diameter of the rotor holes. When it increases from 0.1 to 0.15 m (by 50%), the amplitude of cross section oscillation increases more than 2 times. When the number of holes is increased by 2 times (from 4 to 8), the amplitude of cross section oscillation increases by approximately 2.5 times. But in addition, changing the number of holes and diameter significantly changes the phase of oscillation of the cross-sectional area of the modulator. There is a change in the frequency of oscillations of the cross-sectional area of the modulator. That is why the synchronization of the oscillatory and rotational movements of the PD rotor with VR will depend on the number and diameter of the holes. This is an important conclusion, because it is the correct synchronization of the rotary and oscillating movements of the rotor and crank that will have a significant impact on the energy efficiency and quality of PD emulsion processing with VR.

5 Conclusions

Determination of the change in the cross-sectional area of the modulator of the rotary-pulsation device is the basis for calculating the parameters of such a device: quality of processing, energy consumption and design for a given performance. The change in the cross-sectional area of the RPD modulator is usually represented in the form of a piecewise function – a set of equations at different time intervals, which significantly increases the complexity of the mathematical description of the process. When describing this function for the pulsation device with the vibrating rotor, due to the use of round holes and the equality of their number in the rotor and stator, it became possible to use a continuous function with satisfactory accuracy. In this study, the equations of the dependence of the cross-sectional area on time and the angle of rotation of the crank had been found. Such a representation of the function greatly simplifies the further mathematical description of the process of fluid movement in the interrupter of the device. The equations have no restrictions on the size of the gap between the rotor and the stator and can be used in the calculation of the pulsation device with the vibrating rotor with any gaps between the rotor and the stator.

The obtained functions are necessary in determining such important characteristics of the pulsation device with the vibrating rotor as the instantaneous speed, acceleration and degree of dispersity of the fat emulsion after processing, as well as power and energy consumption. This device makes it possible to use the advantages of resonance of emulsion pulsations to improve the quality of emulsion processing and reduce energy consumption. The equations obtained in this way are the basis for further research and calculations of this promising type of rotary-pulsation devices.

References

1. O. Yatsenko, N. Yushchenko, U. Kuzmyk, V. Pasichnyi, O. Kochubei-Lytvynenko, N. Frolova, O. Korablova, I. Mykoliv, V. Voitsekhivskyi, *Research of milk fat oxidation processes during storage of butter pastes*. Potravinarstvo Slovak J. of Food Sciences, **14**, 443-450. (2020) doi: 10.5219/1283.
2. P. Dhankhar, *Homogenization fundamentals,* IOSR J. Eng. **4**, 1–8 (2014). doi:10.9790/3021-04540108
3. L. Tartar,The General Theory of Homogenization; Lecture Notes. Springer: Berlin/Heidelberg, Germany, **470** (2009)
4. E. Haponiuk, L. Zander, G. Probola, *Effect of the homogenization process on the rheological properties of food emulsions.* Pol. J. Nat. Sci. **30**, 149–158 (2015)
5. N.V. Trusova, I.A.Kohut, S.A. Osypenko, N.G. Radchenko, N.N. Rubtsova, *Implementation of the results of fiscal decentralization of Ukraine and the countries of the European union.* Journal of Advanced Research in Law and Economics **10** (6), 1649–1663 (2019) doi: 10.14505/jarle.v10.6(44).07
6. V. Bulgakov, V. Kuvachov, J. Olt, *Theoretical study on power performance of agricultural gantry systems.* Ann. DAAAM Proc. Int. DAAAM Symp. **30**, 167–175 (2019). doi: 10.2507/30th.daaam.proceedings.022
7. A. Håkanssona, L. Fuchs, F. Innings, F. Laszlo, B. Bergenstahl, C. Tragardh, *Visual observations and acoustic measurements of cavitation in an experimental model of a high-pressure homogenizers*. Journal of Food Engineering **100** (3), 504–513 (2010) doi:10.1016/j.jfoodeng.2010.04.038
8. A. Yong, M. Islam, N. Hasan, *The Effect of pH and High-Pressure Homogenization on Droplet Size.* Sigma J. Eng. Nat. Sci. **35**, 1–22 (2017) doi: 10.26776/IJEMM.02.04.2017.05
9. D. Valencia-Flores, M. Hernández-Herrero, B. Guamis, V. Ferragut, C*omparing the Effects of Ultra-High-Pressure Homogenization and Conventional Thermal Treatments on the Microbiological, Phys, and Chem Quality of Almond Beverages.* J. Food Sci. **78**, 199–205 (2013) doi: 10.1111/1750 3841.12029

10. F. Innings, C. Trägårdh, *Visualization of the drop deformation and break-up process in a high pressure disperser.* Chem. Eng. Technol. **28**, 882–891 (2005) doi:10.1002/ceat.200500080
11. T. Huppertz, Homogenization of Milk / Other Types of Homogenizer (High-Speed Mixing, Ultrasonics, Microfluidizers, Membrane Emulsification), Encyclopedia of Dairy Sciences (Second Edition), 761–764 (2011). doi: 10.1016/B978-0-12-374407-4.00226-0
12. C. Ciron, V. Gee, A. Kelly, M. Auty, *Comparison of the effects of high-pressure microfluidization and conventional homogenization of milk on particle size, water retention and texture of non-fat and low-fat yoghurts.* Int. Dairy J. **20**, 314–320 (2010) doi: 10.1016/j.idairyj.2009.11.018
13. K. Ward, Z. H. Fan, *Mixing in Microfluidic Devices and Enhancement Methods.* J. Micromech. Microeng, **25** (9), 94001–94017 (2015) doi:10.1088/0960-1317/25/9/094001
14. C. Fonte, D. Fletcher, P. Guichardon, J. Aubin, *Simulation of micromixing in a T-mixer under laminar flow conditions,* Chem. Eng. Sci. **222**, 115706 (2020) doi: 10.1016/j.ces.2020.115706
15. C. Lee, C. Chang, Y. Wang, L. Fu, *Microfluidic Mixing: A Review.* Ijms **12** (5), 3263–3287 (2011) doi:10.3390/ijms12053263
16. F. Innings, C. Trägårdh, *Visualization of the drop deformation and break-up process in a high pressure disperser*. Chem. Eng. Technol. **28**, 882–891 (2005) doi:10.1002/ceat.200500080
17. K. Samoichuk, D. Zhuravel, N. Palyanichka, V. Oleksiienko, S. Petrychenko, N. Slobodyanyuk, O. Shanina, I. Galyasnyj, L. Adamchuk, V. Sukhenko, *Improving the quality of milk dispersion in a counter-jet homogenizer.* Potravinarstvo Slovak Journal of Food Sciences **14**, 633-640 (2020) doi: 10.5219/1407
18. K. Samoichuk, D. Zhuravel, O. Viunyk, D. Milko, A. Bondar, Y. Sukhenko, V. Sukhenko, L. Adamchuk, S. Denisenko, *Research on milk homogenization in the stream homogenizer with separate cream feeding.* Potravinarstvo Slovak Journal of Food Sciences **14**, 142–148 (2020) doi:10.5219/1289
19. H. Delmas, L. Barthe, 25 – Ultrasonic mixing, homogenization, and emulsification in food processing and other applications. Power Ultrasonics Applications of High-Intensity Ultrasound 757–791 (2015) doi: 10.1016/B978-1-78242-028-6.00025-9.
20. V. Mohammadi, M. Ghasemi-Varnamkhasti, R. Ebrahimi, M. Abbasvali, *Ultrasonic techniques for the milk production industry. Meas*. J. Int. Meas. Confed. MSRMD **58**, 93–102 (2014) doi: 10.1016/j.measurement.2014.08.022
21. C. Liu, M. Li, C. Liang, W. Wang, *Measurement and analysis of bimodal drop size distribution in a rotor–stator homogenizer.* Chemical Engineering Science **102**, 622–631 (2013) doi:10.1016/j.ces.2013.08.030.
22. L. Di Marzo, P. Cree, D. Barbano, *Prediction of fat globule particle size in homogenized milk using Fourier transform mid-infrared spectra.* J. Dairy Sci. **99**, 8549–8560 (2016) doi: 10.3168/jds.2016–11284
23. M. Rayner, P. Dejmek, Engineering Aspects of Emulsification and Homogenization in the Food Industry; CRC Press, Taylor & Francis Group 322 (2015). ISBN 9781466580435 doi: 10.1201/b18436.
24. M. A. Promtov, M. X. Monastirsky, *Dynamic of cavitational bubbles in rotor impuls apparatus* Journal of Qingdao Just of Chemistry Technical **21**, 4, 318–321 (2000)

25. P. Droździel, T. Vitenko, V. Voroshchuk, S. Narizhnyy, O. Snizhko, *Discrete-impulse energy supply in milk and dairy product processing,* Materials (Basel), **14(15),** 4181 (2021) doi: 10.3390/ma14154181
26. K. Samoichuk, S. Kiurchev, V. Oleksiienko, N. Palyanichka, V. Verholantseva, *Research into milk homogenization in the pulsation machine with a vibrating rotor.* Eastern-European Journal of Enterprise Technologies **6, 11** (84), 16–21 (2016) doi:10.15587/1729-4061.2016.86974
27. A. Panchenko, A. Voloshina, I. Panchenko, O. Titova, A. Caldare*, Design of Hydraulic Mechatronic Systems with Specified Output Characteristics. Advances in Design, Simulation and Manufacturing III*. DSMIE 2020. Lecture Notes in Mechanical Engineering 42–51 (2020) doi: 10.1007/978-3-030-50491-5_5
28. A. Voloshina, A. Panchenko, O. Boltyansky, O. Titova, Improvement of Manufacture Workability for Distribution Systems of Planetary Hydraulic Machines. In: Ivanov V. et al. (eds) Advances in Design, Simulation and Manufacturing II. DSMIE 2019. Lecture Notes in Mechanical Engineering. Springer, Cham., 732–741 (2020) doi:10.1007/978-3-030-22365-6_73
29. S. Ivanovs, V. Bulgakov, H. Kaletnik, L. Shymko, V. Kuvachov, Y. Ihnatiev, *Experimental checking of mathematical models describing the functioning adequacy of bridge systems in agricultural track system.* INMATEH-Agricultural Engineering, **62**(3), 107–114 (2020) doi: 10.35633/inmateh-62-11
30. A. Håkansson, L. Fuchs, F. Innings, J. Revstedt, C. Trägårdh, B. Bergenståhl, *Velocity measurements of turbulent two-phase flow in a high-pressure homogenizer model.* Chem. Eng. Commun. **200**, 93–114 (2013) doi:10.1080/00986445.2012.691921
31. X. Wang, Y. Wang, F. Li, L. Li, X. Ge, S. Zhang, T. Qiu, Scale-up of microreactor: *Effects of hydrodynamic diameter on liquid–liquid flow and mass transfer*. Chem. Eng. Sci. **226**, 115838 (2020) doi: 10.1016/j.ces.2020.115838
32. J. Morales, A. Watts, J. McConville, *Mechanical particle-size reduction techniques.* AAPS Adv. Pharm. Sci. **22**, 165–213 (2016) doi: 10.1007/978-3-319-42609-9_4
33. Y. Liao, D. A. Lucas, *Literature review of theoretical models for drop and bubble breakup in turbulent dispersions.* Chem. Eng. Sci. **64**, 3389–3406 (2009) doi: 10.1016/J.CES.2009.04.026
34. A. Postelmans, B. Aernouts, J. Jordens, T. Van Gerven, W. Saeys, *Milk homogenization monitoring: Fat globule size estimation from scattering spectra of milk*. Innov. Food Sci. Emerg. Technol. **60**, 102311 (2020) doi: 10.1016/j.ifset.2020.102311
35. S. Sharov, A. Zemlianskyi, T. Sharova, H. Viktor, *Ukrainian MOOC: Quantitative and Thematic Analysis of Online Courses, International Journal on Advanced Science,* Engineering and Information Technology **11**, 1143-1149 (2021) doi: 10.18517/ijaseit.11.3.13705
36. V. Bulgakov, V. Kuvachov, J. Olt, *Theoretical study on power performance of agricultural gantry systems*. Ann. DAAAM Proc. Int. DAAAM Symp. **30**, 167–175 (2019) doi: 10.2507/30th.daaam.proceedings.022
37. S. Acharyaa, V. Mishrab, J. Patelc, *Enhancing the mixing process of two miscible fluids: A review.* AIP Conference Proceedings **2341**, 030025 (2021) doi: 10.1063/5.0051818