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Procedia Structural Integrity 51 (2023) 69–75

Structural Integrity Procedia

www.elsevier.com/locate/procedia

6th International Conference on Structural Integrity and Durability (ICSID 2022)

On energy release rate for an electrically permeable interface crack between two different 1D hexagonal piezoelectric QCs

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Abstract

A bi-material composed of two 1D piezoelectric hexagonal quasicrystals with a crack along the material interface was considered. Phonon and phason remote loading providing plane strain conditions were applied at infinity and an open crack model was adopted. Because electromechanical fields have an oscillating singularity at the crack tip, the energy release rate (ERR) is the most important fracture parameter in this case. The main purpose of this study is to define ERR in an analytical form. Using the obtained earlier asymptotic presentations of the phonon and phason fields at the crack tip and the crack closure integral approach, the desired formula is obtained in a simple form. The eligibility of the formula is verified by means of a crack in the bi-material composed of two 1D piezoelectric hexagonal quasicrystals and by considering a crack in homogeneous materials of this type. An additional comparison of the results obtained for a particular case of an isotropic material with known values confirmed the validity of the obtained formula.

Keywords: Piezoelectric quasicrystals; interface crack; energy release rate; phonon and phason fields

1. Introduction

The quasi-crystalline materials, found by Shechtman et al. (1984) are nowadays extensively used in various

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10.1016/j.prostr.2023.10.069

branches of engineering and technology. The mathematical foundations of elasticity theory of quasicrystals (QCs) are presented in Fan (2016). Depending on the atom arrangement 1D, 2D and 3D quasicrystals can be considered (Steurer, Deloudi, 2009). Also, QC bi-materials with piezoelectric effect are used in smart structures.

Different defects like holes, cracks and inclusions are the main reason of failure of quasicrystal devices. Three-dimensional cracks in one-dimensional hexagonal piezoelectric quasicrystals were studied in Fan et al. (2016), and a penny-shaped dielectric crack in the quasicrystal plate of the same structure was considered in Zhou, Li (2019). Two asymmetrical limited permeable cracks emanating from an elliptical hole in one-dimensional hexagonal piezoelectric quasicrystals were considered in Yang et al. (2017).

It is worth mentioning that interface cracks in bi-material and multi-material components are the main cause of failure. The comprehensive review of interface cracks investigation in piezoelectric materials has been done in Govorukha et al. (2016) for tension loading and in Govorukha et al. (2015) for compressive one. However, the cracks between different QC materials have not been sufficiently studied till now. To our knowledge, an arbitrarily shaped electrically impermeable interface crack in a one-dimensional hexagonal thermo-electro-elastic quasicrystal bi-material was investigated in Zhao et al. (2017a, 2017b) by analytically-numerical method, and a plane problem for an electrically permeable interface crack in a 1D piezoelectric QC was studied analytically in Loboda et al. (2020). Besides, several articles related to anti-plane case of an interface crack in QC were recently published.

A crack between dissimilar one-dimensional hexagonal piezoelectric quasicrystals with electrically permeable and impermeable conditions at the crack faces under anti-plane shear and in-plane electric loadings was investigated in Hu et al. (2019). Single and pair interface cracks with mixed conducting-permeable electric conditions in 1D piezoelectric quasi-crystalline space under the action of out of plane phonon and phason shear stresses and in-plane electric field were analytically considered in Loboda et al. (2021) and Loboda et al. (2022), respectively. The problem of multiple collinear electrically permeable interface cracks between dissimilar one-dimensional hexagonal quasicrystals with piezoelectric effect under anti-plane shear and in-plane electric loading was studied in Hu et al. (2021).

The energetic approach to the analysis of 1D hexagonal QCs has not been developed sufficiently till now. We can mention on this subject the paper Sladek et al. (2015) in which path-independent integrals for crack problems in a homogeneous quasicrystal were derived. The importance of the energetic approach to an interface crack in QC is much larger than for a crack in a homogeneous case because of oscillating singularity of near-tip fields and impossibility of introducing the stress intensity factors in a conventional manner. Just the analytical determination of ERR for an interface crack in a piezoelectric QC is the main purpose of the present paper.

2. Formulation of the problem and analytic solution

Consider the plane problem in $x_1 - x_3$ plane for a crack $-a \le x_1 \le a$, $x_3 = 0$ in the interface between two semi-infinite 1D piezoelectric hexagonal quasi-crystalline spaces with point group 6 mm (Fig. 1).

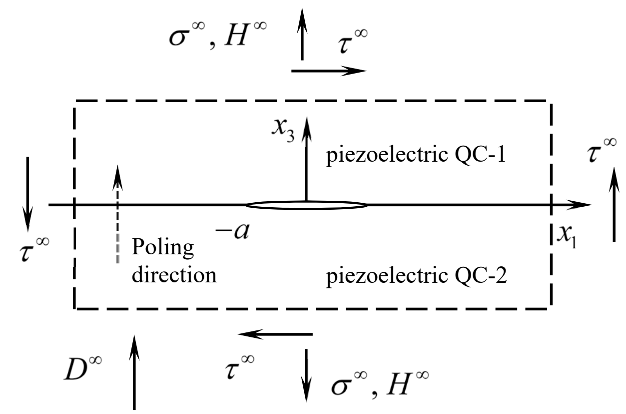

Fig. 1. A crack between two 1D piezoelectric QCs.

The constitutive relations referred to the Cartesian coordinate (x_1, x_2, x_3) with $(x_1, 0, x_2)$ coincident with the periodic plane and x_3-axis identical to the quasi-periodic direction have the form

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{33} \\ \sigma_{13} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{13} & 0 \\ c_{13} & c_{33} & 0 \\ 0 & 0 & 2c_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{33} \\ \varepsilon_{13} \end{Bmatrix} - \begin{bmatrix} 0 & e_{31} \\ 0 & e_{33} \\ e_{15} & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix} + \begin{bmatrix} 0 & R_1 \\ 0 & R_2 \\ R_3 & 0 \end{bmatrix} \begin{Bmatrix} W_{31} \\ W_{33} \end{Bmatrix}, \tag{1}$$

$$\begin{Bmatrix} D_1 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 2e_{15} \\ e_{31} & e_{33} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{33} \\ \varepsilon_{13} \end{Bmatrix} + \begin{bmatrix} \xi_{11} & 0 \\ 0 & \xi_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix} + \begin{bmatrix} \tilde{e}_{15} & 0 \\ 0 & \tilde{e}_{33} \end{bmatrix} \begin{Bmatrix} W_{31} \\ W_{33} \end{Bmatrix}, \tag{2}$$

$$\begin{Bmatrix} H_{31} \\ H_{33} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 2R_3 \\ R_1 & R_2 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{33} \\ \varepsilon_{13} \end{Bmatrix} + \begin{bmatrix} K_2 & 0 \\ 0 & K_1 \end{bmatrix} \begin{Bmatrix} W_{31} \\ W_{33} \end{Bmatrix} - \begin{bmatrix} \tilde{e}_{15} & 0 \\ 0 & \tilde{e}_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix}. \tag{3}$$

The equilibrium equations and geometric equations are the following

$$\sigma_{11,1} + \sigma_{13,3} = 0\,,\ \sigma_{31,1} + \sigma_{33,3} = 0\,,\ D_{1,1} + D_{3,3} = 0\,,\ H_{31,1} + H_{33,3} = 0\,, \tag{4}$$

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i})\,,\ E_i = -\varphi_{,i}\,,\ W_{3i} = W_{3,i}\,, \tag{5}$$

where $i, j = 1, 3$ and the denotation "," represents the derivative operation for the space variables; u_i, W_3 and φ are the phonon displacements, phason displacement, and electric potential, respectively; the atom arrangement is periodic in the $x_1 - x_2$ plane and quasi-periodic in the x_3-axis; σ_{ij} and ε_{ij} are the phonon stresses and strains, respectively; H_{3i} and W_{3i} are the phason stresses and strains, respectively; D_i and E_i are the electric displacements and electric fields, respectively; the polarization direction is along the x_3-axis; c_{ij} and K_i are the elastic constants in the phonon and phason fields, respectively; R_i represent the phonon–phason coupling elastic constants; e_{jk} and $\tilde{e}_{jk}$ are the piezoelectric constants in the phonon and phason fields, respectively; ξ_{ii} are the permittivity constants.

It is assumed that the crack is electrically permeable and uniformly distributed phonon $(\sigma^\infty, \tau^\infty)$ and phason H^∞ stresses as well as electrical displacement D^∞ are prescribed at infinity. The interface conditions can be written in the following form

$$\sigma_{13}^{\pm} = 0\,,\ \sigma_{33}^{\pm} = 0\,,\ H_{33}^{\pm} = 0\,,\ \langle \varphi \rangle = 0\,,\ \langle D_3 \rangle = 0 \ \text{ for } -a < x_1 < a\,, \tag{6}$$

$$\langle \sigma_{13} \rangle = 0\,,\ \langle \sigma_{33} \rangle = 0\,,\ \langle H_{33} \rangle = 0\,,\ \langle u_1 \rangle = 0\,,\ \langle u_3 \rangle = 0\,,\ \langle W_3 \rangle = 0\,,\ \langle \varphi \rangle = 0\,,\ \langle D_3 \rangle = 0 \ \text{ for } x_1 \notin (-a, a)\,, \tag{7}$$

where $\langle f \rangle$ means the jump of the function f over the material interface.

In Loboda et al. (2020) the following presentations were obtained

$$\sigma_{33}^{(1)}(x_1, 0) + m_{j5} H_{33}^{(1)}(x_1, 0) + i m_{j1} \sigma_{13}^{(1)}(x_1, 0) = \Theta_j^+(x_1) + \gamma_j \Theta_j^-(x_1)\,, \tag{8}$$

$$n_{j1} \langle u_1'(x_1) \rangle + i n_{j3} \langle u_3'(x_1) \rangle + i n_{j5} \langle W_3'(x_1) \rangle = \Theta_j^+(x_1) - \Theta_j^-(x_1)\,, \tag{9}$$

where $\Theta_j(z)$ are the functions analytic in the whole complex plane except the crack region $\left(x_1 \in (-b,b),\ x_3 = 0\right)$; $m_{j5} = S_{j5}$, $m_{j1} = -iS_{j1}$, $n_{j1} = Y_{j1}$, $n_{j3} = -iY_{j3}$, $n_{j5} = -iY_{j5}$, $\mathbf{Y}_j = \mathbf{S}_j\boldsymbol{\rho}$; $j = 1,3,5$; γ_j and $\mathbf{S}_j^T = [S_{j1}, S_{j3}, S_{j5}]$ are, respectively, the eigenvalues and eigenvectors of the matrix $(\gamma\,\boldsymbol{\rho}^T + \overline{\boldsymbol{\rho}}^T)$ and $\boldsymbol{\rho}$ is constructed of the matrix $\mathbf{G}$ of dimension 5×5 by crossing out the second and fourth rows and columns; $\mathbf{G} = \mathbf{B}^{(1)}\mathbf{D}^{-1}$, $\mathbf{D} = \mathbf{A}^{(1)} - \overline{\mathbf{A}}^{(2)}(\overline{\mathbf{B}}^{(2)})^{-1}\mathbf{B}^{(1)}$, $\mathbf{A}^{(m)}$, $\mathbf{B}^{(m)}$ are matrixes similar to eponymous matrixes defined in Suo et al. (1992) ($m = 1$ corresponds to the upper material and $m = 2$ – to the lower one).

When obtaining relations (8) and (9), the continuity of stresses, electric displacements and electric potential along the whole material interface were taken into account. Due to this fact fourth row and column were excluded from the matrix $\mathbf{G}$ and also the case $j = 4$ from Eqs. (8) and (9).

Due to (8) the conditions at infinity for the function $\Theta_j(z)$ can be written in the form

$$\Theta_j(z)\big|_{z\to\infty} = (1+\gamma_j)^{-1}(im_{j1}\tau^\infty + \sigma^\infty + m_{j5}H^\infty) \tag{10}$$

Satisfying the interface conditions (6) by using Eq. (8), one arrives at the following problem of the linear relationship:

$$\Theta_j^+\left(x_1\right) + \gamma_j\,\Theta_j^-\left(x_1\right) = 0 \ \text{ for } \ x_1 \in \left(-a,a\right), \tag{11}$$

with the conditions at infinity (10).

According to Muskhelishvili (1975) the solution of this problem has the following form

$$\Theta_j(z) = X_j(z)(\sigma_j^* - i\tau_j^*)(z - 2ib\varepsilon_j), \tag{12}$$

where $X_j(z) = (z+a)^{-1/2+i\varepsilon_j}(z-a)^{-1/2-i\varepsilon_j}$, $\sigma_j^* = \dfrac{1}{r_j}\left(\sigma^\infty + m_{j5}H^\infty\right)$, $\tau_j^* = -m_{j1}\tau^\infty / r_j$, $r_j = (1+\gamma_j)$, $\varepsilon_j = \dfrac{\ln\gamma_j}{2\pi}$ $\left(j = 1,3,5\right)$.

Substituting the solution (12) into the conjugate of Eq. (8) and integrated (9) we get

$$\sigma_{33}^{(1)}(x_1,0) + m_{15}H_{33}^{(1)}(x_1,0) - im_{11}\sigma_{13}^{(1)}(x_1,0) = J_1(x_1), \tag{13}$$

$$\sigma_{33}^{(1)}(x_1,0) + m_{55}H_{33}^{(1)}(x_1,0) = J_5(x_1), \ \text{for} \ x_1 \notin (-a,a) \tag{14}$$

$$n_{11}\left\langle u_1(x_1)\right\rangle + in_{13}\left\langle u_3(x_1)\right\rangle + in_{15}\left\langle W_3(x_1)\right\rangle = \omega_1(x_1), \tag{15}$$

$$n_{53}\left\langle u_3(x_1)\right\rangle + n_{55}\left\langle W_3(x_1)\right\rangle = \omega_5(x_1) \ \text{ for } \ x_1 \in (-a,a). \tag{16}$$

The expressions of $J_1(x_1)$, $J_5(x_1)$, $\omega_1(x_1)$ and $\omega_5(x_1)$ are found with use of (12), but here only their asymptotic expressions are important and they are as follows:

$$J_1(x_1)\big|_{x_1\to a+0} = (q_{11} + iq_{12})\left(x_1 - a\right)^{-0.5+i\varepsilon_1}, \ J_5(x_1)\big|_{x_1\to a+0} = q_3 / \sqrt{x_1 - a}, \tag{17}$$

$$\omega_1\left(x_1\right)\big|_{x_1\to a-0} = \left(q_{21} + iq_{22}\right)\left(a - x_1\right)^{0.5-i\varepsilon_1}, \ \omega_5\left(x_1\right)\big|_{x_1\to a-0} = q_4\sqrt{a - x_1}, \tag{18}$$

where

$$q_{11} + iq_{12} = \sqrt{\frac{a}{2}}(1+\gamma_1)\left[\left(d_1\cos\xi_1 + d_2\sin\xi_1\right) + i\left(-d_1\sin\xi_1 + d_2\cos\xi_1\right)\right],\ q_3 = \sqrt{2a}\,\sigma_5^*, \tag{19}$$

$$d_1 = \sigma_1^* - 2\varepsilon_1\tau_1^*,\ d_2 = 2\varepsilon_1\sigma_1^* + \tau_1^*,\ \xi_1 = \varepsilon_1\ln(2a).$$

$$q_{21} + iq_{22} = \frac{1+\gamma_1}{\sqrt{\gamma_1}}\left[\left(\tau_1^*\cos\xi_1 - \sigma_1^*\sin\xi_1\right) + i\left(\sigma_1^*\cos\xi_1 + \tau_1^*\sin\xi_1\right)\right]\sqrt{2a},\ q_4 = 2\sqrt{2a}\sigma_5^*. \tag{20}$$

The phonon and phason stresses and displacements at the crack tip a can be found from the systems (13), (14) and (15), (16), respectively.

3. The energy release rate (ERR)

According to the crack closure integral (Rybicki, Kanninen, 1977), the energy release rate (ERR) at a crack tip a can be presented in the form

$$G_a = \lim_{\Delta l\to 0}\frac{1}{2\Delta l}\left\{\int_a^{a+\Delta l}\left[\sigma_{33}^{(1)}(x_1,0)\left\langle u_3(x_1-\Delta l)\right\rangle + \sigma_{13}^{(1)}(x_1,0)\left\langle u_1(x_1-\Delta l)\right\rangle + \right.\right.$$

$$\left.\left. + H_{33}^{(1)}(x_1,0)\left\langle W_3(x_1-\Delta l)\right\rangle + D_3^{(1)}(x_1,0)\left\langle \varphi(x_1-\Delta l,0)\right\rangle\right]dx_1\right\}. \tag{21}$$

The last term in the integrand is equal to zero because of zero electric potential jump in the crack region. Substituting the above mentioned phonon and phason asymptotic expressions into (21) and performing the integration we arrive to the following formula for the ERR:

$$G_a = \frac{\pi\left(1+4\varepsilon_1^2\right)}{8\cosh(\pi\varepsilon_1)}h_1h_2 + a\pi h_3\left(\sigma_5^*\right)^2, \tag{22}$$

where

$$h_1 = \frac{m_{55}n_{55}+n_{53}}{p_1p_2} - \frac{1}{m_{11}n_{11}},\ h_2 = q_{11}q_{22} - q_{12}q_{21},\ h_3 = \frac{m_{15}n_{15}+n_{13}}{p_1p_2},\ p_1 = m_{55} - m_{15},\ p_2 = n_{13}n_{55} - n_{15}n_{53}.$$

4. The numerical illustration and discussion

Numerical analysis was performed for the bi-material composed of the piezoelectric QCs with the characteristics given bellow (Zhang et al., 2016).

The upper material:

elastic constants (GPa): $c_{11} = 150$, $c_{12} = 100$, $c_{13} = 90$, $c_{33} = 130$, $c_{44} = 50$, $K_1 = 0.18$, $K_2 = 0.3$, $R_1 = -1.50$, $R_2 = 1.20$, $R_3 = 1.20$;

piezoelectric constants ($\mathrm{C\,m^{-2}}$): $e_{31} = \tilde{e}_{15} = -0.160$, $e_{33} = 0.347$, $e_{15} = -0.138$, $\tilde{e}_{33} = 0.350$;

dielectric constants ($10^{-9}\mathrm{C^2\,N^{-1}m^{-2}}$): $\xi_{11} = 0.0826$, $\xi_{33} = 0.0903$. (22)

The lower material:
elastic constants (GPa): $c_{11} = 234.33$, $c_{12} = 57.41$, $c_{13} = 66.63$, $c_{33} = 232.22$, $c_{44} = 70.19$, $K_1 = 122$, $K_2 = 24$, $R_1 = R_2 = R_3 = 0.8846$;
piezoelectric constants ($\mathrm{C\,m^{-2}}$): $e_{31} = -4.4$, $e_{33} = 18.6$, $e_{15} = 11.6$, $\tilde{e}_{15} = 1.16$, $\tilde{e}_{33} = 1.86$;
dielectric constants ($10^{-9}\mathrm{C^2\,N^{-1}m^{-2}}$): $\xi_{11} = 5$, $\xi_{33} = 10$. (23)

The variation of the ERR at the right crack tip for $\sigma^{\infty} = 1$ MPa , $H^{\infty} = 0$, $a = 100$ mm and different values of the remote shear stress τ^{∞} are presented in Fig. 2. Line *I* corresponds to the bi-material mentioned above, lines *II* and *III* are drawn for the particular cases of the homogeneous material with the characteristics equal to the lower and upper materials, respectively. It can be seen that the ERR for the composite material are approximately equal to the mean value of the energy release rates for the homogeneous materials which characteristics are equal to those of each components of the composite.

The similar variation of the ERR for $\sigma^{\infty} = 1$ MPa , $\tau^{\infty} = 0$, $a = 100$ mm and different values of H^{∞} are drawn in Fig. 3. Line *I* corresponds to the bi-materials (22) and (23), lines *II* and *III* are obtained for the same cases of the homogeneous material as in Fig. 2. It is interesting to mention that usually the values of ERR increase with increasing of H^{∞} . However, this growing is rather weak in the whole considered interval for the homogeneous material (22) (line *II*), it is almost constant in the interval $(0, 0.5)$ for the bi-material case (line *I*) and the ERR even decreases at the initial section of the considered region for the homogeneous material (23) (line *III*).

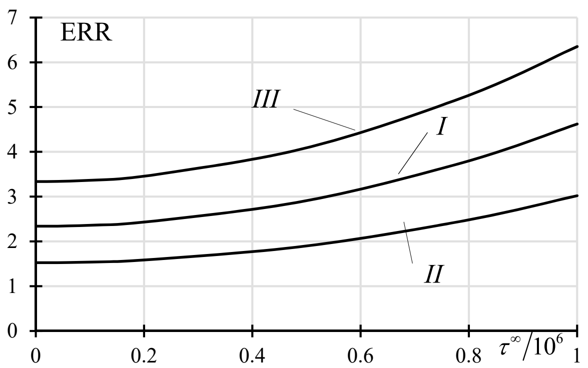

Fig. 2. The variation of the ERR N/m with respect to the remote phonon shear stress τ^{∞} (Pa).

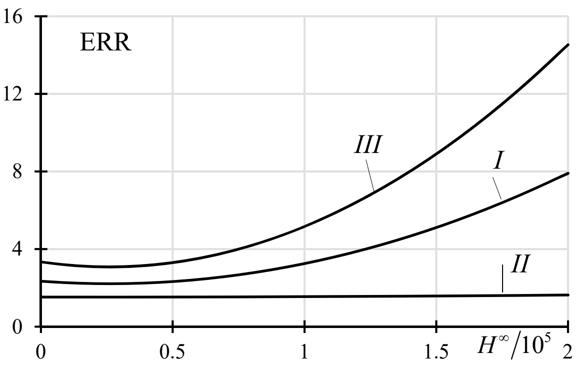

Fig. 3. The variation of the ERR (N/m) with respect to the phason stress H^{∞} (Pa).

To confirm the correctness of the obtained results we found the ERR for the case of homogeneous QC with characteristics (23), in which we assumed $c_{13} = 94$ GPa , $c_{33} = 234.33$ GPa , $R_1 = R_2 = R_3 = 0$ and all piezoelectric constants approximately equal to 0. In this case we got the uncoupled isotropic material with Young's modulus $E \approx 180$ GPa and Poison's ratio $\nu \approx 0.286$. The calculation of the ERR for such material with $\sigma^{\infty} = 1$ MPa , $\tau^{\infty} = 0$, $H^{\infty} = 0$ and $a = 100$ mm gives $G = 1.598$ N/m . This value completely coincides with the known analytical result obtained on the formula $\left(1-\nu^2\right)\pi a\left(\sigma^{\infty}\right)^2 / E$. By the way the value of the ERR for this loading and homogeneous QC with characteristics (23) is 1.527 N/m (see line *II* of Fig. 2).

5. Conclusions

The problem of electrically permeable crack between dissimilar 1D hexagonal quasicrystals with piezoelectric effect is considered. Mixed mode phonon and phason loading and electric displacement can be prescribed remotely from the crack. The main attention is devoted to the analytical determination of the energy release rate at the crack tip. For this purpose the analytically obtained asymptotic expressions of electromechanical components at the vicinity of the crack tip together with the crack closure integral approach are used. The energy release rate is found exactly in the form of very simple formula (22). With use of this formula the ERR for the bi-material composed of two specific quasicrystals is found with respect to the external loading variation. Besides, the values of the ERR for homogeneous quasicrystals are also found, and comparison of the obtained results is made. It is particularly shown that the ERR for the composite material are approximately equal to the mean value of the energy release rates for the homogeneous materials, the characteristics of which are equal to those of individual components of the composite. Conformation of the validity of the obtained formula (22) for the ERR is done by means of consideration of particular case of equivalent characteristics of the upper and lower materials, the piezoelectric and coupling phonon-phason constants of which are equal to zero and, moreover, the phonon material constants correspond to isotropic material. The excellent agreement of the ERR in this case with known analytical result is discovered.

Acknowledgements

The authors are grateful the 2021 "Belt and Road" innovative talent exchange foreign expert project. No. DL2021003002 and the Ministry of Education and Science of Ukraine, project. No. 0121U109767.

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